THERMOSYPHON ANALYSIS OF A CANDU REACTOR CORE IN THE EVENT OF LOSS-OF-FLOW ACCIDENTS
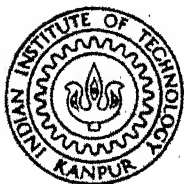
By

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NUCLEAR ENGINEERING AND TECHNOLOGY PROGRAMME

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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THERMOSYPHON ANALYSIS OF A CANDU REACTOR CORE IN THE EVENT OF LOSS-OF-FLOW ACCIDENTS

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

By

SUCHANDAN KUMAR DAS

to the

TO

MY

BELOVED   PARENTS
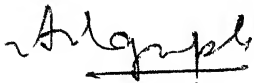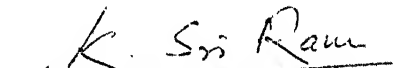
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CERTIFICATE

This is to certify that this work entitled,
" THERMOSYPHON ANALYSIS OF A CANDU REACTOR CORE IN THE
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Das has been carried out under our supervision and has not
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SYNOPSIS

In this study, hydraulic and thermosyphon models are formulated to investigate the transient and steady state flow and, thermal behaviour of a CANDU Reactor core in the event of loss-of-flow accidents, resulting from pumping failures. The hydraulic analysis leads to a flow coastdown, which determines mass flow rate versus time. The transient temperature behaviour is evaluated next. In view of safety assessment following the transients, major attention is directed towards the performance of thermosyphoning (Natural convection) effects in the vicinity of a horizontal cylindrical fuel rod to extract the after heat due to the decay of fission products, under both isothermal and non-isothermal surface conditions on the fuel rod. It is found that natural convection Nusselt number is approximately one-fifteenth of the forced convection (for coolant velocity 27.2 ft/sec) value. Furthermore, the surface temperature variation has a significant effect on the heat transfer from the fuel rod.

CHAPTER I

INTRODUCTION

1.1 SAFETY OF POWER REACTORS

The thorough safety assessment of a nuclear power
plant necessarily involves consideration of the potentiality
and consequences of a large number of reactor accidents,
ranging from minor mishaps to truely catastrophic events. In
recent years the efforts associated with assuring the safety
of large nuclear power reactors has accelerated rapidly. The
specifics of the accidents sequences that must receive atten-
tion depend strongly on reactors concept and on the details of
design and operation of heat transport system, fission product
barriers and safety systems. Among broad categories of reactor
accidents, loss-of-flow accidents (LOFA) encompass one of the
most conceivable nuclear power plant accidents.

Loss-of-flow accidents can result either from pumping
failures or from obstruction in coolant loops. Flow obstruc-
tions are most likely to lead to local flow starvation in the
core, whereas pumping failures are most likely to affect a
major part or all of the core. Pumps, however, are normally
in active state of operation and they are subject to rapid
failure either from mechanical seizure or from power failure.
A catastrophic mechanical seizure of pumps may occur nearly

instantaneously, causing a precipitous drop of the pump head
to zero. But, pump failure due to loss of power does not result
in an instantaneous loss of head, since even if pump motor
suddenly fails, the rotational inertia of the pump impeller and
flywheel cause the pump to coastdown gradually over a period of
time that is often of the order of several seconds.

In loss-of-flow accidents, caused by a power failure
to the all coolant pumps, the pressure drop across the core
decreases, causing decreased mass flow rates and increased core
temperature. Flow starvation may also take place, if there is
an increase in the core hydraulic loss coefficient. Flow
decrease results in deterioration of the heat transfer coeffi-
cient and coolant flow rate. As a result, the elevation of
the coolant temperature may lead to a boiling crisis and subse-
quent insulation of the fuel element from the coolant. The
equilibrium flow following the transient is quite small, since
it is driven only by natural circulation. Thus, the most
important action to be taken in the majority accident situations
is the prompt shutdown of the nuclear chain reaction. This
action rapidly reduces the power to the level of decay heat and
therefore greatly alleviates the energy dissipation problem
under adverse condition. .

Once reactor shutdown is achieved immediately, the
primary problem is to assure an orderly path of removal of

decay heat. Otherwise, the energy imbalance would eventually

lend to the meltdown of core and cause penetration of the

primary system. Thus, a major engineered safety feature of any

nuclear power plant is the emergency core cooling system.

Equally important, in the event of a loss-of-flow accident is

that, continuous core cooling must also be provided over a period

of time required to actuate emergency core cooling system or in

situations when emergency core cooling is not immediately

available. The rate of decay heat production during this period

after shutdown may still be capable of melting fuel elements.

Under such circumstances, thermosyphon system is

extremely important and capable of transmitting farely large

rates of heat transfer in the earths gravitational field. It

determines amount of core cooling immediately available, even

if the driving pressure following the transient fails to sus-

tain natural circulation of coolant in the primary flow circuit.

The reactor core consists of a number of flow channels, acts as

a system of thermosyphons and provides essential cooling

required during such emergent situations. A study of the thermo-

syphon systems reveals the nature of physical phenomena associa-

ted with the process of heat transfer.

1.2 THERMOSYPHON SYSTEM

The heat transfer system which has become known as

the thermosyphon has many applications in engineering, ranging

from its' use in Bakers' oven to the cooling of turbine rotor
blades, internal combustion engines and nuclear reactors. A
thermosyphon is a circulating fluid system, whose motions are
caused by density differences in a body force field. Davis
and Morris [1] have suggested that thermosyphon can be catego-
rized according to (i) nature of boundaries (whether the system
is open or closed to mass flow), (ii) the regime of heat transfer
(whether the process is purely natural convection or mixed
natural and forced convection) and (iii) the nature of the body
force (gravitational or rotational). The thermosyphon system
has an intrinsic function of removing heat from a prescribed
source and transferring heat and mass over a specific path
(frequently a recirculation flow) and rejecting heat and mass
to a prescribed sink. These flows are intrinsically driven by
thermal buoyancy forces either locally or in overall sense. A
simple loop flow may be the result of local buoyancy forces,
but a multibranched flow circuit may easily incorporate sections
in which the flow direction is contrary to the local buoyancy
forces resulting from the pressure created by the overall system
buoyancy forces.

The classification of thermosyphon systems mainly
includes:
(1) A common single phase, natural convection open system in
the form of a vertical tube open at the top and closed at the
bottom.

(2)   A simple single phase natural convection closed system in
      the form of a vertical tube closed at both ends.

(3)   The single phase closed loop thermosyphon.

Vertical Open Thermosyphon :

        The vertical open thermosyphon as shown in Fig. 1.1
provides the starting point for considering thermosyphon systems.
As can be noted that, the primary effect of heating the wall of
an open thermosyphon should be to cause some type of flow upward
along the wall due to the buoyancy effects and an associated
return flow downward in the core via the law of continuity.   It
has been found that for large heat fluxes on the wall (as may be
the case of nuclear reactors), the buoyancy forces are suffi-
ciently intense near the wall so that a boundary layer regime
is obtained.   On the other hand for weaker heat fluxes, the
buoyancy forces are less and the effect of shear is relatively
enhanced owing to small inertial effects, causing the boundary
layer to try to fill the entire tube.   Hence, compared to the
ordinary boundary layer flow, the flow is impended.   For suffi-
ciently weak fluxes this effect is found to be rather uniform
and a similarity flow is observed.   The first experiments on an
open vertical thermosyphon were reported by Foyle [2], but those
were of limited extent.   His observations show that the heat
transfer was independent of tube length beyond a certain critical
value although, it increases quickly with increasing diameter
of the tube.   He also pointed out that, for increasing values of
length to diameter ratio (L/a), larger values of Rayleigh number
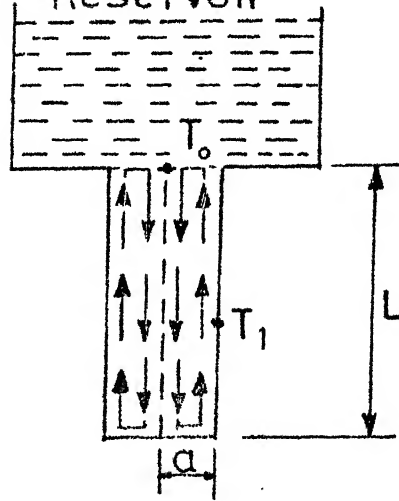
Fig.1·1 Vertical open thermosyphon
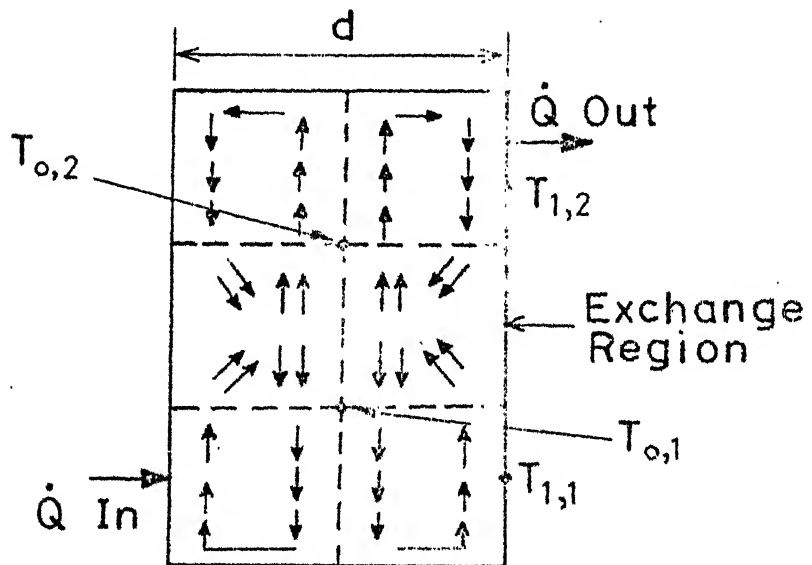
Fig.1·2 Vertical closed thermosyphon
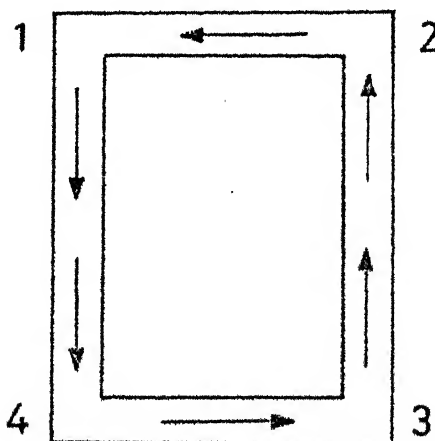
are required to attain any given heat flux level.

Vertical Closed Thermosyphon :

The simple closed vertical thermosyphon as shown in Fig. 1.2 gained popularity as the problem of containment and pressurization became apparent in various application of open thermosyphon. It consists of a fluid completely enclosed in a tube situated in a body force field and lying parallel to the direction of the body force. Heat is applied to the 'lower' part of its length and cooled over the upper part, thus producing flow patterns as shown in Fig. 1.2. Refering to the figure, the first subscripts 0 and 1 associated with the temperature indicate core region and wall respectively. The second subscripts 1 and 2 refer to the top and bottom region of the thermosyphon. $\dot{Q}$ gives the heatflow. It has been found that, when closed thermosyphon modelled carefully, it can be treated as a synthesis of two simple open thermosyphons approximately, joined at the mid tube exchange (coupling) region. Most of the modes of flow found for the open thermosyphon have been found for the closed thermosyphons. Primary problem concerned with the closed thermosyphon is that of modelling the exchange region i.e. to find temperatures T_{01} and T_{02}, to apply open thermosyphon results. For lower values of temperature difference between the bottom and upper wall, the exchange mechanism was found to be basically convective and for higher values, the convective process is less stable and degenerates to a mixing exchange process.

Closed Loop Thermosyphon :

A simple closed loop thermosyphon shown in Fig. 1.3 is a similar geometric configuration, which can be found in many industrial situations. The virtue which made closed loop thermosyphon interesting is that, it avoids chocking that occurs in open thermosyphon and diminishes exchange process as occurs in open and closed thermosyphons. The core boundary layer interaction is also minimized. In principle, there is no limit to the types of flow that could be obtained in a closed loop thermosyphon, subject to various thermal, geometric and body force conditions. Under the condition of rotation of this configuration, for a given temperature difference between the wall and the coolant, gives a strong increase to the heat transfer coefficient. The better known application of closed loop thermosyphon, is cooling of internal combustion engines and nuclear reactors, when driving pressure establishes a natural circulation.

1.3 REVIEW OF LITERATURE

The vertical thermosyphon systems and its various applications in engineering has been the subject of a good deal of research in recent years, although, probably because of the complexities of thermal buoyancy forces and their dependence on geometrical configurations and flow regimes, little attention has been given to it in its horizontal form. More attention has been directed towards the open thermosyphon in which the heated lower region opens at the top into a large reservoir of fluid

from which heat is removed by allowing evaporation to occur.
Cohen and Baley [3], reported a number of such experiments
using both static and rotating apparatus, concentrating mainly
upon the evaporative system. The original application of this
system to turbine cooling was given by Schmidt [4]. Experimental
investigation to remove heat from the reservoir by means of an
immersed cooling coil was carried out by Cohen and Hartnett
et al. [5,6]. Under these conditions, it was found that, the
fluid returning from the reservoir has free access to the "core"
of the thermosyphon.

The analytical study presented by Lighthill [7] has
served as the foundation for most vertical thermosyphon analyses
(laminar) upto the present time. His treatment of flow in the
open thermosyphon yielded three laminar flow regimes, depending
upon wall heat fluxes. He considered basic laminar boundary
layer flow equations for an open circular thermosyphon with
variable fluid properties as follows :

$$\frac{\partial(\rho u R)}{\partial x} + \frac{\partial(\rho v R)}{\partial R} = 0 \qquad (1.3.1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial R} = -g - \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{1}{\rho R}\frac{\partial}{\partial R}\left(R\mu\frac{\partial u}{\partial R}\right) \qquad (1.3.2)$$

$$\frac{\partial P}{\partial R} = 0 \qquad (1.3.3)$$

$$u\frac{\partial(C_p T)}{\partial x} + v\frac{\partial(C_p T)}{\partial R} = \frac{1}{\rho R}\frac{\partial}{\partial R}\left(R K \frac{\partial T}{\partial R}\right) \qquad (1.3.4)$$

$$\rho_1 = \rho[1 + \beta(T - T_1)] \qquad (1.3.5)$$

where, u, v are velocities in x and R directions respectively, g is gravitational acceleration. K, P, β, ρ, C_p and μ are thermal conductivity, pressure, coefficient of thermal expansion, density, specific heat and dynamic viscosity respectively. Subscript 1 refers to the bottom of the thermosyphon.

With these equations, he treated the case of Pr = ∞, and thus neglecting the inertial terms after non-dimensionalizing the equations and estimated that, the analyses would give only 10 % error for Pr = 2. For different values of wall heat fluxes at the bottom of the thermosyphon, he used Blasius-Howarth Series solution to solve the problem. He finally established theoritically the existence of boundary layer flow regimes for large heat fluxes, impended flow regimes for weaker heat fluxes and similarity flow regimes for sufficiently weak fluxes. Martins' [8] experimental results provided an excellent verification of Lighthills' flow models. His local heat transfer results for lower values of wall heat flux also verified the existence of laminar impended regimes and similarity regimes.

Lighthill [7] again considered that the closed vertical thermosyphon system could be treated as a synthesis of open systems coupled together hydrodynamically in some manner by a region of limited extent. He suggested that complete mixing of the opposite streams in the coupling region would probably be a reasonable approximation. But it was left to Bayley and Lock [9], who made the first comprehensive experimental and analytical study of the closed thermosyphon. Their attempt was to analyse

the system for the laminar boundary layer regime. They found
that, in each section of the tube a thin laminar boundary layer
was formed adjacent to the tube wall, thus producing a central
core of fluid which moves slowly in the opposite direction.
Three limiting possibilities were suggested for the manner in
which the open systems were coupled together depending upon the
fluid properties. Those were :

(1) The violent collision of the axially converging boundary
     layers giving rise to vigorous mixing.
(2) The non-interactive crossing over of the boundary layers,
     each becoming the core of the opposite section.
(3) The return of each boundary layer to its own section;
     thermal energy is thus transferred across the coupling
     region by axial conduction only.

They theoretically treated the above mentioned hydro-
dynamic couplings of the mid tube exchange mechanism and deter-
mined the effect of Prandtl number on heat transfer rates.
Considering the integral form of the energy and momentum equation
as cited below,

$$\mathcal{V}\left(\frac{\partial u}{\partial y}\right)_0 = \beta g \delta \int_0^1 (T-T_1)\,d(y/\delta) - \frac{d}{dx}\delta \int_0^1 u^2 d(y/\delta) \qquad (1.3.6)$$

$$K\left(\frac{\partial T}{\partial y}\right)_0 = -\frac{d}{dx}\delta \int_0^1 u(T-T_1)\,d(y/\delta) \qquad (1.3.7)$$

where, δ is the boundary layer thickness. Other notations have
their usual meaning and subscripts 'o' and 'l' refer to wall and
core respectively.

The approach developed by them permitted solutions to be obtained for vertical tubes of almost any cross section and including all fluid for which $1 < Pr < \infty$. The method is based on the assumptions that, the two dimensional form of the boundary layer equations are approximately valid near the tube wall and using Vonkarman -- Pohlhausen technique, they obtained the following expressions.

$$(\bar{N}u_d)_H = 0.677 (\frac{Pr(t_d)_H}{Pr + 20/21})^{1/4} \times \left\{ 1 - \frac{1}{0.382(\frac{Pr(t_d)_H}{Pr+20/21})^{1/4} - 2} \right.$$

(1.3.8)

for an isothermal wall and,

$$(\bar{N}u_d)_H = 0.862(\frac{Pr(t_d)_H}{Pr + 4/5})^{1/4} \times \left\{ 1 - \frac{1}{0.410(\frac{Pr(t_d)_H}{Pr + 4/5}) - 2} \right\}^{1/4}$$

(1.3.9)

for a uniformay heated wall. Subscripts 'd' and 'H' refer to the diameter and heated section respectively and t_d is the modified Rayleigh number.

Japikse and Winter [10] pointed out that, the wall temperature on each tube half should be used to model the flow processes in that tube half. This is of considerable importance to avoid error in heat transfer rates, in situations where non-isothermal wall condition exists.

However, the application of the above studies, mostly dealing with "vertical" thermosyphons are basically confined only to Light Water Reactors (LWR). Boiling Water Reactors (BWR) a

specific case of LWRs, have a vertical core configuration and easily actuate vertical thermosyphon cooling processes in the event of flow failure accidents. On the other hand, thermosyphoning for fluid enclosed in a horizontal tube or channels situated in a body force field and lying perpendicular to the direction of the body force on a horizontal plane, is known very little. Thus, for "horizontal" reactor core configuration e.g. CANDU Reactor (RAPP), this aspect is of considerable importance to assess the safety features involved in thermosyphon cooling in flow failure accidents.

No literature, however, found available, explicitly treating horizontal thermosyphon systems in the gravitational force field. Since, thermosyphoning is closely associated with natural convective heat transfer processes, some insight can be gained by reviewing natural convection processes around the horizontal objects.

For laminar flow around horizontal cylinders, Hyman et al. [11] found that the following equation for the average Nusselt number correlated data well for water, tolune, silicone, mercury etc.

$$\bar{N}u_D = 0.53[(\frac{Pr}{Pr + 0.952}) (Gr_D \cdot Pr)]^{1/4} \qquad (1.3.10)$$

In some cases of horizontal cylinder, the entire surface may not be parallel to the body force caused by gravity. The variations of the angle between the surface and gravity vector was considered by Herman [12], who derived an equation for the local Nusselt number at various positions around a horizontal cylinder for air

$(Pr = 0.74)$. The equation is

$$Nu_D = 0.604 \, Gr_D^{1/4} \, \phi(\theta) \qquad (1.3.11)$$

The function $\phi(\theta)$ depends upon the value of θ, measured from the bottom of the cylinder. An integral approach to this problem by Levy [13] and a similarity solution approach by Eckert and Soehngen [14], also agree well with the calculated values by Herman.

On the basis of the foregoing reviews, the prime objective in the present investigation is to determine, thermo-syphoning effects, to provide immediate core cooling in Reactors of horizontal core systems in the event of loss-of-flow accidents. A typical example is the CANDU (RAPP) type Reactor.

1.4 DESCRIPTION OF CANDU REACTOR SYSTEM

CANDU is a heavy water moderated pressurized heavy water cooled, Uo_2 fuelled pressure tube reactor. The reactor has a horizontal cylindrical tank and tube assembly. This assembly is known as calandria. Natural Uranium in the form of pressed pellets is sheathed in Zircaloy tubes to form cylindrical fuel elements. Nineteen such elements are assembled to form a fuel bundle. Calandria tube contains thick walled coolant tube having a fuel bundle and primary coolant i.e., heavy water at high pressure. The calandria shell contains heavy water at high pressure and it is used as a moderator.

Heavy water essentially serves two purposes, namely as a moderator and as a coolant, each having its own closed
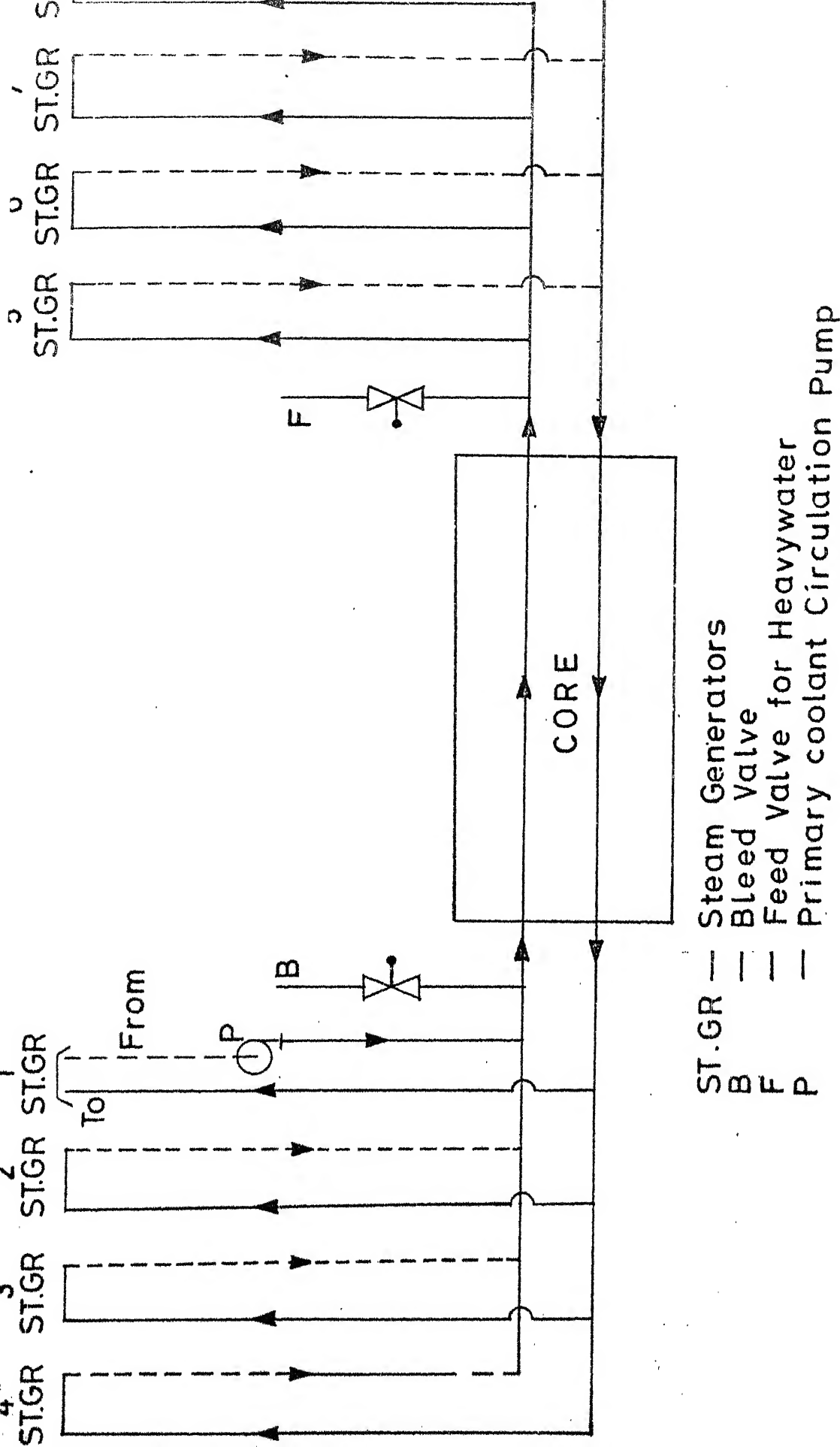
Fig.1.4 Schematic Diagram for Primary Coolant Circuit

ST.GR — Steam Generators
B — Bleed Valve
F — Feed Valve for Heavywater
P — Primary coolant Circulation Pump

circulating system. The fuel coolant system is called primary
heat transport system is a high pressure and high temperature
circuit with no bulk boiling. But, the moderator circuit is a
low temperature low pressure circuit. Both circuits are insula-
ted from each other in the reactor core by a gas annulas between
the calandria tube and coolant tube.

Heat generated in the fuel elements is transferred to
heat exchangers in the primary heat transport system, where the
heat is utilized to generate steam in a secondary coolant system,
consists essentially of two identical loops in series, one at
each end of the reactor. Four boiler assemblies and four pumps
in parallel are included in each loop. The primary coolant is
circulated through alternate coolant tube assemblies in opposite
directions. Each boiler assembly consists of ten vertical
U type tubes and shell heat exchangers in parallel and a steam
drum. The hot heavy water from the reactor enters the tube
sides of the heat exchangers at the bottom of their recircula-
ting sections. Then it returns from the bottom of the preheater
legs of the heat exchangers to the reactor. Schematic diagram
of the primary flow circuit is shown in Fig. 1.4.

The secondary coolant system consists of boiler feed-
water entering the shell side of each heat exchanger at the bottom
of the preheat leg. The feedwater is heated near to the satura-
tion temperature. As a result, it boils in the upper portion of
the leg and enters the steam drum above it. Water and steam are
separated here. The water at saturation temperature flows through

downcomers to the bottom of heat exchanger recirculating leg,
where it boils and returns to the steam drum. Saturated steam
is supplied to the turbine unit. Discharge valves are provided
in the steam mains to release steam to dump condenser in the
case of pressure rise in steam drum. Such a facility is provi-
ded, in the case turbine is unable to accept enough steam. The
turbine and reactor have adequate controls for safety and smooth
operation of the power plant.

1.5 SCOPE OF THE THESIS

Scope of the present work is mainly centered on the
investigation of thermosyphoning effects in CANDU Reactor System.
It determines amount of core cooling immediately available as an
essential safety requirement in loss-of-flow accidents particu-
larly, in situations, when emergency core cooling is not imme-
diately available and equilibrium driving pressure ceases to
establish a natural circulation in the primary circuit. However,
transient flow and temperature behaviours are also essential
features of a loss-of-flow accident and must be evaluated prior
to steady state analysis.

Under operating conditions the bulk of the fission
product inventory remains trapped in the fuel. The fission
products that escape from the fuel consists primarily of fraction
of noble gases and other species of more volatile fission product
These collect in the fuel-cladding gap and in other void regions
on the cladding envelope. Their existence may affect the gap

conductance and give rise to non-isothermal cladding surface conditions, resulting in a variation of steady state heat transfer across the surface.

In the investigation proposed here, the objectives will thus be to :

(a) Study the flow coastdown during the transient, which determines decrease in mass flow rate with time.

(b) Find out whether the driving pressure, following the total loss of pumping power can establish a natural circulation equilibrium flow in the primary circuit.

(c) Determine the temperature distribution of the core during the transient and approximate evaluation of the time required to attain steady state.

(d) Evaluate the important thermosyphoning (steady state natural convection) effect on the flow and heat transfer in terms of the distributions of velocity and temperature in the adjacent boundary layer regime of fuel elements, considering both isothermal and non-isothermal cladding surface conditions. Cases of linear and non-linear variations of surface temperatures are treated in non-isothermal conditions.

Obviously, detailed analysis of a project of this magnitude would be a lengthy one and requiring much time and effort. The analysis will thus be carried out only as far as is required to arrive at a qualitative conclusion for the objectives listed in items from (a) to (c). In the perspective of

thermosyphoning, listed in item (d), an asymptotic similarity
solution of the equations of continuity, conservation of momentum
and energy in the boundary layer regime will be sought by
numerical methods for an axisymmetric, constant property, laminar
boundary layer flow of a viscous, incompressible and newtonian
fluid around a horizontal cylindrical fuel element. There is an
arbitrary distribution of temperature on the surface of the fuel
rod. Gravitational force field generates buoyancy forces
responsible for thermosyphoning in the adjacent boundary layer.

    For ease in formulation and analysis of the present
problem as envisaged in the preceeding objectives, those may be
mainly divided into two parts. The first part named as " Flow
modelling', comprises first two objectives (a) and (b), which
deals with transient and steady state flow behaviours in the
primary system and the second part namely, " Thermal modelling"
comprises rest two objectives (c) and (d), deals with transient
temperature distribution in the core followed by thermosyphon
cooling.

CHAPTER II

FLOW MODELLING

The determination of both transient and steady state flow behaviour following the transient, associated with reactor accidents, due to common mode power failure to pumps requires formulation of a hydraulic model for the reactor core and primary circuit. The mass flow rate versus time must be determined to gain an insight into the flow coastdown phenomena. Following the transient, equilibrium flow is quite small and driven only by natural circulation. It is required to verify that whether natural circulation can be sustained in the primary circuit.

2.1 HYDRAULIC TRANSIENT ANALYSIS

To accomplish objective (a) of Sec. 1.5, transient hydraulic analysis is formulated. Refering to Bird et al. [15] and Lewis [16]; control volume form of the continuity and mechanical energy balance equations in Eulerian frame are utilized to arrive at a relationship between mass flow rate and pressure drop for a flow channel.

Mass and mechanical energy balance equations are written as follows:

$$\frac{\partial}{\partial t} \iiint_{c.v.} \rho \, dV + \iint_{c.s.} \rho \, \hat{n}.\vec{v} \, ds = 0 \qquad (2.1.1)$$

$$\frac{\partial}{\partial t} \iiint\limits_{c.v.} (\tfrac{1}{2}\rho v^2)\ dV + \iint\limits_{c.s.} (\tfrac{1}{2}\rho v^2)\ \hat{n}.\vec{v}\ ds$$

$$= - \iiint\limits_{c.v.} \vec{v} \cdot \nabla P\ dV + \iiint\limits_{c.v.} \vec{v}.\vec{g}\ dV - E_v + E_p \qquad (2.1.2.)$$

where, ρ = fluid density, V = fluid volume, s = surface of the
volume V, $\hat{n}$ = outward normal to s, v = fluid speed, $\vec{v}$ = fluid
velocity, P = pressure, $\vec{g}$ = gravitational force vector,
E_v = Rate of mechanical energy dissipation due to viscous force
and E_p = Rate of mechanical energy generation due to pump.
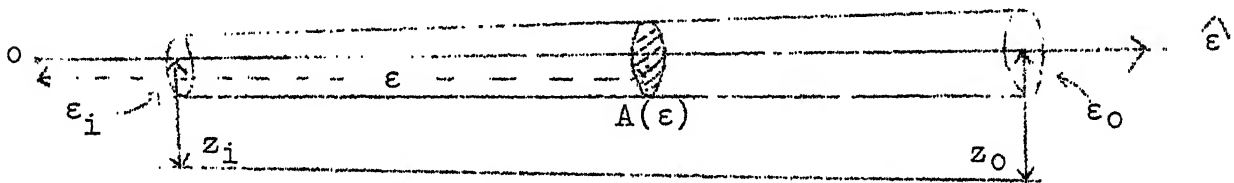
Fig. 2.1 : Flow channel.

Let us consider a specific flow channel as shown in
Fig.(2.1). ε is the distance along the centreline and $dV = A(\varepsilon)d\varepsilon$,
where $A(\varepsilon)$ is channel area at ε. Fluid velocity, speed, density
and pressure are considered function of ε and time.

$$v = v(\varepsilon,t), \quad v = \vec{v}(\varepsilon,t)\hat{\varepsilon}$$

$$\rho = \rho(\varepsilon,t), \quad P = P(\varepsilon,t)$$

Considering the volume V between an inlet ε_i and outlet ε_o,
equations (2.1.1) and (2.1.2) may be written as

$$\frac{\partial}{\partial t} \int_{}^{\varepsilon_o}$$

$$\frac{\partial}{\partial t} \int_{\varepsilon_i}^{\varepsilon_o} \frac{1}{2} \rho(\varepsilon,t)v^2(\varepsilon,t)A(\varepsilon)d\varepsilon + \frac{1}{2}\rho(\varepsilon_o,t)v(\varepsilon_o,t)^3 A(\varepsilon)$$

$$- \frac{1}{2}\rho(\varepsilon_i,t)v(\varepsilon_i,t)^3 A(\varepsilon_i)$$

$$= - \int_{\varepsilon_i}^{\varepsilon_o} v(\varepsilon,t)A(\varepsilon)\frac{\partial P(\varepsilon,t)}{\partial \varepsilon}d\varepsilon - g\int_{\varepsilon_i}^{\varepsilon_o}\rho(\varepsilon,t)v_z(\varepsilon,t)A(\varepsilon)d\varepsilon - E_v + E$$

$$(2.1.4)$$

where, g is the magnitude of gravity vector and v_z vertical component of v.

In the absence of boiling, we can treat primary system flow transients by assuming incompressible flow, i.e. $\frac{\partial \rho}{\partial t} = 0$, which means that $\rho = \rho(\varepsilon)$. Time independence of ρ causes the first term of equation (2.1.3) to vanish, and we have

$$\rho(\varepsilon_o)v(\varepsilon_o,t)A(\varepsilon_o) = \rho(\varepsilon_i)v(\varepsilon_i,t)A(\varepsilon_i) \qquad (2.1.5)$$

Since, ε_o is arbitrary, it gives mass flow rate $\rho(\varepsilon)v(\varepsilon,t)A(\varepsilon)$ to be independent of ε. Thus, mass flow rate at any point along the channel is given as

$$W(t) = \rho(\varepsilon) v(\varepsilon,t) A(\varepsilon) \qquad (2.1.6)$$

Using these incompressible flow approximations and carrying out the mathematics involved, the expression for pressure drop follows from equation (2.1.4)

$$\Delta P = \frac{L}{\bar{A}}\frac{dW}{dt} + \left[\left(\frac{\bar{\rho}\bar{A}}{\rho_o A_o}\right)^2 - \left(\frac{\bar{\rho}\bar{A}}{\rho_i A_i}\right)^2\right]\frac{W^2}{2\bar{\rho}\bar{A}^2} + \bar{e}_1\frac{W^2}{2\bar{\rho}\bar{A}^2} + \bar{\rho}g(z_o - z_i) - \bar{\rho}gH_p$$

$$(2.1.7)$$

where, $P = P_i - P_o$, $L = \varepsilon_i - \varepsilon_o$, $\bar{\rho} = \frac{1}{V}\int_{\varepsilon_i}^{\varepsilon_o}\rho(\varepsilon)A(\varepsilon)d\varepsilon$,

$$\frac{1}{A} = \frac{\bar{F}}{L} \int_{\varepsilon_i}^{\varepsilon_0} \frac{d\varepsilon}{\bar{\rho}(\varepsilon)A(\varepsilon)} , \quad \bar{E}_V = \frac{E_V}{W(t)}, \quad H_p = \frac{E_p}{gW(t)} ,$$

e_l, is an empirical loss coefficient and subscripts 'o' and 'i' refer to outlet and inlet respectively.

The sum of contributions to the pressure drop come from inertial, viscous, hydrostatic and pump energy term respectively. Combining, viscous and acceleration pressure drop expressions, eqn. (2.1.7) can be written in a contracted form.

$$\Delta P = \frac{L}{\bar{A}} \frac{dW}{dt} + K \frac{W^2}{2\bar{\rho}\bar{A}^2} + \bar{\rho}g \Delta z - \bar{\rho}g H_p \qquad (2.1.8)$$

where, $K = \bar{e}_l + (\frac{\bar{\rho}\bar{A}}{\rho_o A_o})^2 - (\frac{\bar{\rho}\bar{A}}{\rho_i A_i})^2$, known as hydraulic resistance of the channel, and $\Delta z = z_o - z_i$.

2.2 TREATMENT OF THE PRIMARY FLOW CIRCUIT

The standard pressure drop and flow relationship given in equation (2.1.8) can be applied to CANDU system. Considering a simplified assumption that all pumps behave identically following a power failure and pump inertia following the loss of power is taken into account in the expression for pressure drop.

With the above assumption, the accident can be treated in terms of the reactor core and N identical coolant loops (The loop consists of series of paths through the hot and cold legs and the heat exchangers). Core is characterized by a loss coefficient K_c and flow area A_T. Each of the coolant loops has a loss coefficient K_l and flow area A_l. Normalized flow resistance i.e., $\bar{K}_c = K_c/A_T^2$ and $\bar{K}_l = K_l/A_l^2$ are defined to eliminate flow area

from the pressure drop equation.

Excluding hydrostatic term for a horizontal core ($\therefore \Delta z = z_o - z_i = 0$) in the pressure drop relationship, we write core pressure drop as

$$\Delta P_c = \left(\frac{L}{A}\right)_c \frac{dW}{dt} + \bar{K}_c \frac{W^2}{2\bar{P}} \qquad (2.2.1)$$

and for coolant loops

$$\Delta P_1 = \left(\frac{L}{A}\right)_\ell \frac{dW_1}{dt} + \bar{K}_1 \frac{W_1^2}{2\bar{P}} + g(\bar{P}\Delta z)_1 + \bar{P}g\left(\frac{\omega}{\omega_0}\right)^2 H_p \qquad (2.2.2)$$

Where, W_o and W_1 are flow rates in the core and in each flow loop respectively. The term $\left(\frac{\omega}{\omega_0}\right)^2$ multiplies the initial pump head in order to approximate the decreasing speed of the pump during the transient. ω_0 is the initial pump speed and ω is the decreasing speed during the transient.

Applying, Kirchoffs laws for pressure drop around any loop of the channel and hydraulic junction law for flows in a hydraulic circuit, we have :

$$\Delta P_c + \Delta P_1 = 0 \qquad (2.2.3)$$

$$W = NW_1 \qquad (2.2.4)$$

Now, combining expressions (2.2.1) and (2.2.2), we find that the core flow is determined from

$$\left(\frac{L}{A}\right)_p \frac{dW}{dt} + \bar{K}_p \frac{W^2}{2\bar{P}} + g(\bar{P}\Delta z)_p - \bar{P}g\left(\frac{\omega}{\omega_0}\right)^2 H_p = 0 \qquad (2.2.5)$$

where for the primary system,

$$\left(\frac{L}{A}\right)_p = \left(\frac{L}{A}\right)_c + \frac{1}{N}\left(\frac{L}{A}\right)_l$$

$$\bar{K}_p = \bar{K}_c + \frac{1}{N^2} K_l \quad \text{and} \quad (\bar{F}\Delta z)_p = (\bar{F}\Delta z)_l$$

Subscript p refers to primary circuit.

Hydrostatic head is small compared to pump head during the first stage of the transient and can be neglected from equation (2.2.5). It is only predominant after pump head vanishes at a later stage.

Refering to Burgreen [17] to model the pump speed, we write the equation of motion :

$$I \frac{d\omega(t)}{dt} = -c\omega^2(t) \tag{2.2.6}$$

subject to initial condition $\omega(t)\big|_{t=0} = \omega_0$ where, I is the moment of inertia of the pump, c is a loss coefficient relating to windage torque. Solution of (2.3.6) gives

$$\omega = \frac{\omega_0}{1 + t/t_p} \tag{2.2.7}$$

Where, $t_p = I/c\,\omega_0$ is the pump halftime. Substituting equation (2.2.7) in (2.2.5) and neglecting hydrostatic term in accordance with previous argument, we obtain

$$\left(\frac{L}{A}\right)_p \frac{dW}{dt} + \bar{K}_p \frac{W^2}{2\bar{\rho}} = \frac{\bar{\rho}g\,H_p}{(1 + t/t_p)^2} \tag{2.2.8}$$

subject to initial condition

$$K_p \frac{W_0^2}{2\bar{\rho}} - \bar{\rho}g\,H_p = 0 \tag{2.2.9}$$

The nonlinear differential equation can be written in terms
of the normalized core flow utilizing the initial condition, as

$$t_1 \cdot \frac{d}{dt}\left(\frac{W}{W_0}\right) + \left(\frac{W}{W_0}\right)^2 = \frac{1}{(1 + t/t_p)^2} \qquad (2.2.10)$$

where, $t_1 = \dfrac{2\bar{\rho}}{W_0 \bar{K}_p (\frac{A}{L})_p}$ designated as loop halftime.

A solution of equation (2.2.10) gives the flow coast-
down relation in terms of the normalized flow as a function of
time. For CANDU reactors designed with sufficiently large
flywheels, the pump inertia may be made much largers than that
of the fluid, and hence $t_p \gg t_1$. If this is the case, the
normalized flow rate varies much slowly with time and it is
reasonable to neglect the time derivative in equation (2.2.10).
This approximately gives flow coastdown by

$$\frac{W}{W_0} = \frac{1}{1 + t/t_p} \qquad (2.2.11)$$

For a typical recirculation pump, the pump halftime, t_p is
calculated to be 5 seconds. Table I gives numerical results
of variation of normalized flow with respect to time, normalized
with pump halftime, using relation (2.2.11). A flow coastdown
plot of these results is given in Figure (2.2).

When t_1 and t_p are of the same order of magnitude,
a solution of nonlinear equation (2.2.10) must be sought. As
such, no standard method in terms of elementary functions found
applicable to it. Some aspects of the solution is given in
Appendix A.

$$\left(\frac{W}{W_o}\right)\sim\left(\frac{t}{t_p}\right). \text{ Curve}$$

Pump half time $t_p = 5$ sec.

Ratio of the elapsed time to pump half time,
(Dimensionless)
(t/t_p) ———►

Fraction of the initial flow ◄——— (W/W_o)
(Dimensionless)
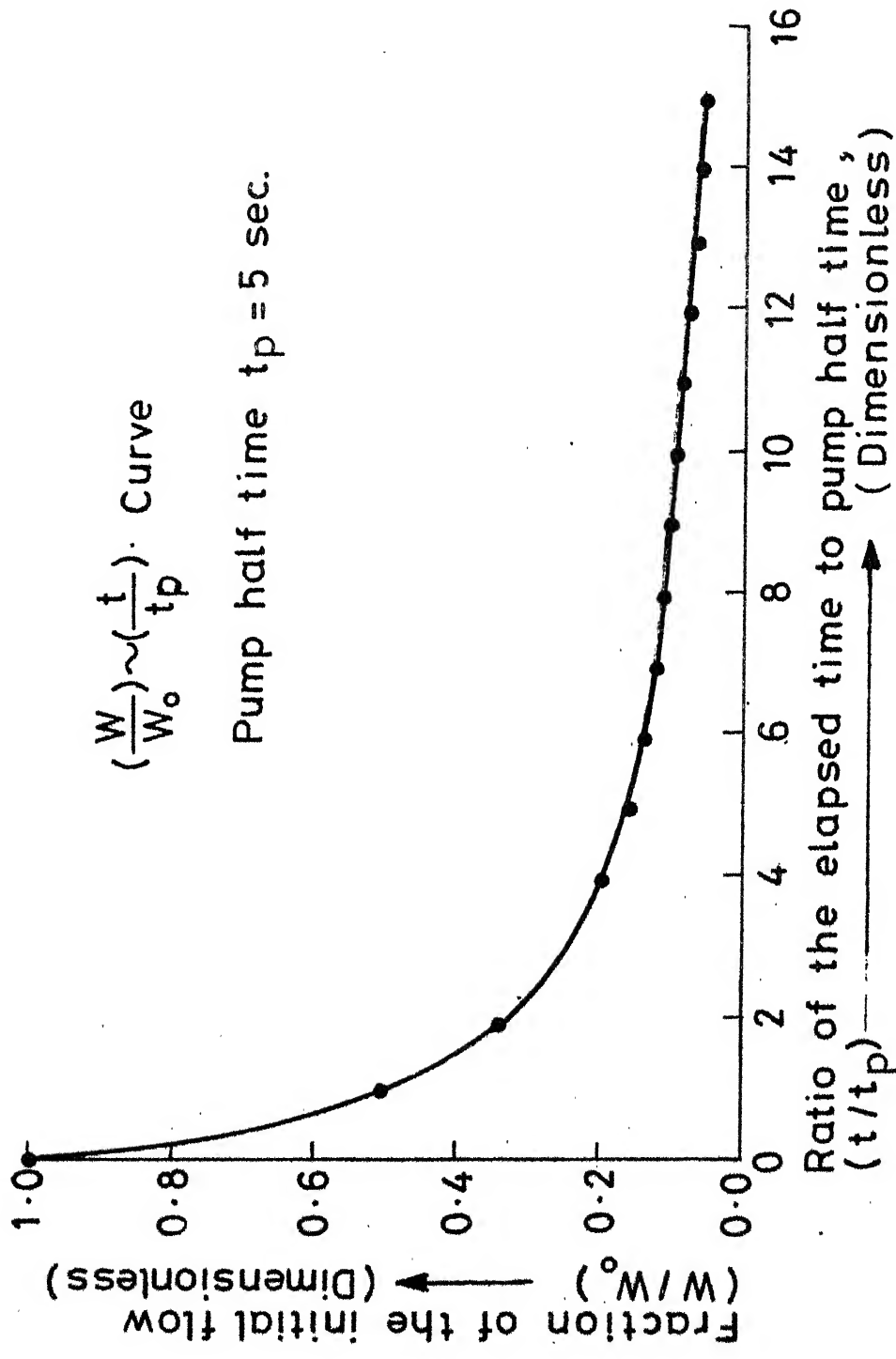
Fig. 2·2 Flow coast-down curve

Table I

(Flow Coastdown Results)

Pump Halftime $= t_p = 5$ sec.

Ratio of the elapsed time to the pump halftime, (t/t_p)	Franction of initial flow, W/W_0 (Normalized flow)
0	1.0
1	0.5
2	0.33
3.	0.25
4	0.20
5	0.16
6	0.14
7	0.12
8	0.11
9	0.10
10	0.09
11	0.08
12	0.076
13	0.071
14	0.066
15	0.062

2.3 DRIVING PRESSURE IN NATURAL CIRCULATION

The second important aspect in flow modelling i.e., objective (b) Sec. 1.5, is to verify whether natural circulation can be sustained in the primary circuit following the total loss of pumping power. Because of the density differential between the coolant in the downcomer and that in the riser, a driving pressure is established. Provided the reactor is shutdown fast enough to prevent any damage, a central question then becomes whether an equilibrium state can be established for natural circulation to remove decay heat.

To establish such an equilibrium state the driving pressure must equal to the total system losses at the desired rate of flow of coolant in the channel. This must be verified. Following El-Wakil [18], the system losses are given by

$$\sum \Delta P = \sum \Delta P_f + \sum \Delta P_a + \sum \Delta P_r \qquad (2.3.1)$$

Where, $\sum \Delta P$ = Total pressure losses in lbf/ft^2,

$\sum \Delta P_f$= Sum of frictional pressure losses, in core riser and downcomer,

$\sum \Delta P_a$ = Sum of the acceleration pressure losses,

$\sum \Delta P_r$ = Sum of the pressure losses due to resistance to flow such as, due to drag of submerged bodies like spacers, fuel element handle etc.

The total system losses found out from RAPP data is $1.413 \ lbf/in^2$. The driving pressure may be evaluated as follows:

$$\Delta P_d = \text{(hydrostatic pressure in downcomer)}$$
$$- \text{(hydrostatic pressure in riser)}$$

$$= \bar{\rho}_d \ H \ \frac{g}{g_c} - \bar{\rho}_r \ H \ \frac{g}{g_c} = (\bar{\rho}_d - \bar{\rho}_r) H \ \frac{g}{g_c} \qquad (2.3.2)$$

where, H is the height of the downcomer as well as the riser and $\bar{\rho}_d$, $\bar{\rho}_r$ are mean densities in the downcomer and riser respectively. g is the gravitational acceleration and g_c is the conversion constant.

In standard gravity $g/g_c = 1.0$. Substituting corresponding values in equation (2.3.2), $\triangle P_d$ is found out to be 26.31 lbf/ft^2 i.e., 0.182 lbf/in^2.

Now, in the case the computed driving pressure being less than the total system losses, the coolant rate of flow decreases and natural circulation cannot be sustained. The flow ultimately comes to a stagnation as the steady state is attained.

CHAPTER III

THERMAL MODELLING

A comprehensive analysis of the problem of thermal
modelling for a reactor core under accident conditions and
steady state immediately following the transient, frequently
requires the numerical solution of large systems of partial
differential equations for transient heat conduction, fluid flow,
transient and steady state convection and a host of other
phenomena likely to occur during and after the course of a
transient. Simple lumped parameter models basing on energy
balances, however, often provides sufficient accuracy when the
primary objectives are to gain an understanding of the inter-
action of various phenomena involved and to make rough estimates
of the consequences of the accident. But, the present formula-
tion of the problem largely revolves around two important aspects,
namely, the transient temperature distributions in the core and
steady state thermosyphoning cooling effects following the
transient, which is of paramount importance inview of safety
assessment.

3.1 TRANSIENT TEMPERATURE DISTRIBUTION

Accurate description of the temperature distribution
that appear in the reactor core under accident conditions
requires that the time dependent mass, momentum and energy
balance conservation equations to such situations, Mayer [20],
must be solved successively by numerical methods. The complexity
in the process of solution is due to the time involvement and

severe nonlinearity   for the presence of convective
terms in the inertial force expression. This can be further
aggravated owing to the reactivity effects through tempera-
ture feedback.

For the present investigation as proposed in the
objective (c) sec. 1.5, we try to formulate a simple trans-
ient conduction analysis to arrive at an expression for the
solution of this equation subject to given boundary
conditions. In the process, we shall deal with the analytical
method to derive a solution, in order to gain an insight
into the procedure of solution for non-homogeneous boundary
conditions. Limited resources of data and inadequate
informations regarding the boundary conditions of the reactor
core during the process of transient, puts a constraint for
arriving at a complete solution of the problem. The analy-
tical treatment will be general, and a solution can be
immediately sought, once the required boundary conditions
are known.

In the case of very severe transients, the
analytical technique may not   provide the desired accuracy
due to large changes in fuel temperature and coolant
conditions. This fact coupled with obvious complexities of
analytical technique and availability of digital computers
has led to an increased use of numerical techniques. The
mathematical apparatus required for the finite difference

solution of parabolic equation of heat conduction type
and required convergence criteria for explicit finite
difference solution was comprehensively treated by Crank
and Nicholson [21].

In treating the problem of transient temperature
distribution in the reactor core by a conduction analysis,
some simplifying assumptions are made.

These are :

1. The radial dimension of the fuel bundle placed in the
    core is small compared to the axial i.e. horizontal
    dimension.

2. While the axial heat flux distribution (i.e. volumetric
    thermal energy) in the core varies sinusoidally, the
    radial distribution shows relatively small changes.
    Thus, the neutron flux and consequently the volumetric
    thermal source strength q''', are considered as functions
    of only the axial coordinate.

3. It will be also assumed that the physical properties
    of the coolant and the fuel (e.g. density, specific
    heat, thermal conductivity) remain constant and
    independent of temperature.

4. Core is considered to be of constant cross-sectional
    dimension.

The sinusoidal core heat flux distribution can be utilized to find out the initial temperature distribution in the core with the help of the boundary conditions at the inlet and outlet prior to the transient. The volumetric thermal energy distribution is shown in Fig. 3.1. The distribution is of sine form with respect to the origin situated at the inlet and positive axial co-ordinate is taken towards the outlet. T_1 and T_2 are the inlet and outlet temperatures respectively, prior to the transient. Their numerical values are quoted in Appendix III. These values refer to the reactor conditions during steady state operation i.e. prior to the initiation of transients.
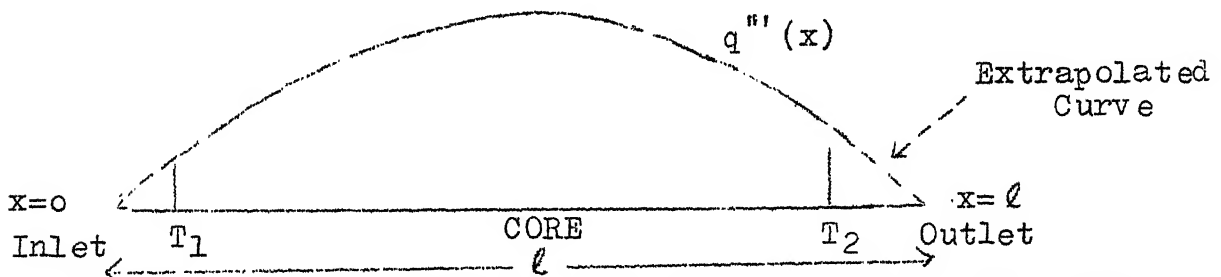
Fig.3.1: Sinusoidal Heat flux distribution in the core.

The total length of the core is considered to be ℓ = 16.404 ft. (This includes extrapolation distance.) Under transient conditions with an initial sinusoidal volumetric thermal energy distribution in the axial direction, the general conduction equation for heat flow considered only in one dimension (i.e. one space variable) assumes the well known form.

$$C_p \rho \frac{\partial T(x,t)}{\partial t} = K \frac{\partial^2 T(x,t)}{\partial x^2} \qquad (3.1.1)$$

where, t represents time, x axial coordinate, C_p specific heat, ρ density and K thermal conductivity and $T(x,t)$ is the temperature distribution. The initial volumetric thermal energy distribution assumes the following form

$$q'''(x) = q_0''' \sin \left(\frac{\pi x}{\ell}\right) \qquad (3.1.2)$$

where, q_0''' is the peak value of the thermal energy inside the core.

The initial and boundary conditions to this boundary value problem is stated as follows:

$$T(0, 0) = T_1$$
$$\text{at } t = 0 \qquad (3.1.3)$$
$$T(\ell, 0) = T_2$$

$$T(0, t) = \alpha_1(t)$$
$$\text{for } t > 0 \qquad (3.1.4)$$
$$T(\ell, t) = \alpha_2(t)$$

A complete solution of the present formulation to evaluate the transient temperature distribution i.e. seeking a particular solution to the conduction equation demands that the functions $\alpha_1(t)$ and $\alpha_2(t)$ should be explicitly

defined as a function of time. No experimental data or theoretical estimates of the nature of functions $\alpha_1(t)$ and $\alpha_2(t)$ during the process of transient is available. However, a general solution to the problem can be sought by analytical method such that the transient temperature behaviour during the course of flow transient can be known, if the functions $\alpha_1(t)$ and $\alpha_2(t)$ are specified.

Initial temperature distribution in the core :

The initial temperature distribution can be evaluated from the steady state conduction equation and thermal energy distribution, which represents a source factor.

Thus, from equation (3.1.1) and (3.1.2) for $t = 0$, we have

$$\frac{d^2 T(x,o)}{dx^2} + \frac{q_o'''}{K} \sin \left(\frac{\pi x}{\ell}\right) = 0 \qquad (3.1.5)$$

Above equation integrates twice to give

$$T(x,o) = C_1 + C_2 x + \left(\frac{\pi}{\ell}\right)^2 \sin \left(\frac{\pi x}{\ell}\right) \qquad (3.1.6)$$

where, C_1 and C_2 are constants of integration.

Initial temperature distribution at any position can be determined by evaluating these integration constants with the help of the conditions enumerated in eqn.(3.1.3).

This gives two simultaneous equations in two unknowns C_1 and C_2.

$$\begin{pmatrix} 1 & 0 \\ 1 & \ell \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix}$$

which is solved to give,

$$C_1 = T_1 \quad \text{and} \quad C_2 = \frac{T_2 - T_1}{\ell}$$

Hence, from equation (3.1.6), the core initial temperature distribution follows as,

$$T(x,o) = T_1 + \left(\frac{T_2 - T_1}{\ell}\right)x + \left(\frac{\pi}{\ell}\right)^2 \sin\left(\frac{\pi x}{\ell}\right) \qquad (3.1.7)$$

Analytical method :

The complete boundary value problem will be:

$$\frac{\partial T(x,t)}{\partial t} = \frac{K}{\rho c_p} \frac{\partial^2 T(x,t)}{\partial x^2}$$

With boundary conditions; and initial conditions :

$$T(x,o) = T_1 + \left(\frac{T_2 - T_1}{\ell}\right)x + \left(\frac{\pi}{\ell}\right)^2 \sin\left(\frac{\pi x}{\ell}\right) = T(x)$$

$$T(o,t) = \alpha_1(t), \quad T(\ell, t) = \alpha_2(t)$$

For non-homogeneous **boundary** conditions prescribed in the given problem, it becomes **necessary** to split up the boundary conditions into groups of partial conditions, and solutions of the differential equations satisfying each group separatel

must be sought. Then, their sum automatically satisfies the heat conduction equation, on accounts of it's linearity. The original division into partial conditions should be carried out in a correct manner such that, the boundary cond: tions will also be satisfied.

The scheme for the decomposition of the boundary conditions and of the complete solution $T(x,t)$ in partial solutions $v(x,t)$ and $w(x,t)$ is carried out as follows.

$$T(x,t) \; = \; v(x,t) + w(x,t) \qquad (3.1.8)$$

Then equation (3.1.1) becomes

$$\frac{\partial v(x,t)}{\partial t} + \frac{\partial w(x,t)}{\partial t} = \frac{K}{\rho C_p} \left[\frac{\partial^2 v(x,t)}{\partial x^2} + \frac{\partial^2 w(x,t)}{\partial x^2} \right] \qquad (3.1.9)$$

The transformed boundary conditions will be :

$$v(o,t) + w(o,t) \; = \; \alpha_1(t)$$
$$v(\ell,t) + w(\ell,t) \; = \; \alpha_2(t) \qquad (3.1.10)$$

and the initial condition becomes,

$$v(x,o) + w(x,o) \; = \; T(x) \qquad (3.1.11)$$

Now, $w(x,t)$ can be chosen such that,

$$\frac{\partial^2 w}{\partial x^2} \; = \; 0$$

$$w(o,t) \; = \; \alpha_1(t) \qquad (3.1.12)$$

$$w(\ell,t) \; = \; \alpha_2(t)$$

Hence, we obtain, $w(x,t) = a(t)x + b(t)$ (3.1.13)

Applying the boundary conditions (3.1.12) we have,

$$b(t) = \alpha_1(t)$$

and

$$a(t) = [\alpha_2(t) - \alpha_1(t)]/\ell$$

Now, $v(x,t)$ satisfies the partial differential equation,

$$\frac{\partial v(x,t)}{\partial t} - \frac{K}{\rho C_p} \frac{\partial^2 v(x,t)}{\partial x^2} = - \frac{\partial w(x,t)}{\partial t}$$ (3.1.14)

where $w(x,t)$ is a known function of x and t.

The initial and boundary conditions becomes

> I.C. $v(x,o) = T(x) - w(x,o) = g(x)$

> B.C. $v(o,t) = o; \quad v(\ell, t) = o$ (3.1.15)

where, $w(x,t)$ is given by,

$$w(x,t) = \frac{1}{\ell} [\alpha_2(t) - \alpha_1(t)]x + \alpha_1(t)$$ (3.1.16)

Therefore,

$$- \frac{\partial w}{\partial t} = - \frac{x}{\ell}(\frac{d\alpha_2(t)}{dt} - \frac{d\alpha_1(t)}{dt}) + \frac{d\alpha_1(t)}{dt}$$

Let us use the notation,

$$f(x,t) = - \frac{x}{\ell} (\frac{d\alpha_2(t)}{dt} - \frac{d\alpha_1(t)}{dt}) + \frac{d\alpha_1(t)}{dt}$$

Thus, our equation for $v(x,t)$ becomes

$$\frac{\partial v(x,t)}{\partial t} = \alpha^2 \frac{\partial^2 v(x,t)}{\partial x^2} + f(x,t)$$ (3.1.17)

where, $\alpha^2 = K/\rho C_p$, with boundary conditions

$$v(o,t) = o , \quad v(\ell,t) = o$$ (3.1.18)

and initial condition

$$v(x,o) = T(x) - w(x,o)$$ (3.1.19)

This is an inhomogeneous equation with homogeneous boundary conditions.

Let us seek a solution to this boundary value problem in the form

$$v(x,t) = \sum_{n=1}^{\infty} T_n(t) \sin \frac{n\pi x}{\ell} \qquad (3.1.20)$$

So that the boundary conditions (3.1.18) should be satisfied. Let us suppose that the function $f(x,t)$, regarded as a function of x, can be expanded in Fourier series.

$$f(x,t) = \sum_{n=1}^{\infty} f_n(t) \sin \frac{n\pi x}{\ell} \qquad (3.1.21)$$

where, $f_n(t) = \frac{2}{\ell} \int_0^{\ell} f(x,t) \sin \frac{n\pi x}{\ell} \, dx$

Substituting the series (3.1.20) and (3.1.21) into equation (3.1.17), we obtain,

$$\left[\frac{dT_n(t)}{dt} - \left(\frac{n\pi\alpha}{\ell}\right)^2 T_n(t) - f_n(t)\right] \sin \frac{n\pi x}{\ell} = 0$$

Let us put $\frac{n\pi\alpha}{\ell} = \beta_n$, we obtain

$$\frac{dT_n(t)}{dt} - \beta_n^2 T_n(t) = f_n(t) \qquad (3.1.22)$$

By using the initial condition for $v(x,t)$

$$v(x,o) = \sum_{n=1}^{\infty} T_n(o) \sin \frac{n\pi x}{\ell} = g(x)$$

We obtain initial condition for $T_n(t)$:

$$T_n(o) = \frac{2}{\ell} \int_0^{\ell} g(x) \sin \frac{n\pi x}{\ell} \, dx$$

The solution of eq. (3.1.22) becomes

$$T_n(t) = e^{\beta_n^2 t} [T_n(o) + \int_0^t e^{-\beta_n^2(t-\tau)} f_n(\tau)d\tau] \quad (3.1.23)$$

Substituting (3.1.23) in (3.1.20) we have, the complete expression for, $v(x,t)$ follows as,

$$v(x,t) = \sum_{n=1}^{\infty} \left\{ e^{\beta_n^2 t}[T_n(o) + \int_0^t e^{-\beta_n^2(t-\tau)} f_n(\tau)d\tau] \right\}$$

$$X \sin\left(\frac{n\pi x}{\ell}\right) \quad (3.1.24)$$

where, $T_n(o) = \dfrac{2}{\ell} \displaystyle\int_0^{\ell} g(x) \sin\left(\frac{n\pi x}{\ell}\right) dx$

Thus, the complete solution to the problem is the combination of both the partial solutions $v(x,t)$ and $w(x,t)$, i.e.

$$T(x,t) = \sum_{n=1}^{\infty} \left\{ e^{\beta_n^2 t} [T_n(o) + \int_0^t e^{-\beta_n^2(t-\tau)} f_n(\tau)d\tau] \right\} \sin\left(\frac{n\pi x}{\ell}\right)$$

$$+ \frac{1}{\ell} [\alpha_2(t) - \alpha_1(t)]x + \alpha_1(t) \quad (3.1.25)$$

Transient Numerical Technique :

When the transients are severe and the transient boundary conditions are of such form that a mathematical solution to the problem is not possible. In these cases, the problems are best handled by a numerical technique with hand calculator or computers depending upon the accuracy desired.

In the setup for the problem the second derivatives in equation (3.1.1) is approximated by

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{(\Delta x)^2}(T^n_{m+1} - 2\,T^n_m + T^n_{m-1})$$

and

$$\frac{\partial T(x,t)}{\partial t} = \frac{T^{n+1}_m - T^n_m}{\Delta t}$$

In this relation, the superscript designates the time increment and subscript corresponds to space increment. Combining the relations above, the difference equation equivalent to Eq.(3.1.1) is

$$\frac{T^n_{m+1} - 2\,T^n_m + T^n_{m-1}}{\Delta x^2} = \frac{1}{\alpha^2}\frac{T^{n+1}_m - T^n_m}{\Delta t} \qquad (3.1.24)$$

Thus, if the temperature of the various nodes are known at any particular time, the temperature after an increment of time Δt may be calculated by writing an equation like eqn.(3.1.24) for each node and obtaining values of T^{n+1}_m. The procedure may be repeated to obtain the distribution after any desired number of time increments. Equation (3.1.24) can be conveniently written as,

$$T^{n+1}_m = \frac{\alpha^2 \Delta t}{\Delta x^2}(T^n_m + T^n_{m-1}) + \left[1 - \frac{2\alpha^2 \Delta t}{\Delta x^2}\right]T^n_m \qquad (3.1.25)$$

In this explicit finite difference method an approximation associated with difference equation (3.1.25) is that, while the temperatures at T^n_m changes during a small interval

to a value T_m^{n+1}, the values of T_{m+1}^n and T_{m-1}^n are assumed to remain constant.

If the time and distance increments are chosen such that $M = \dfrac{(\Delta x)^2}{\alpha^2 \Delta t} = 2$, the temperature of node m after the time increment is given as the arithmetic average of the two adjacent nodal temperatures at the begining of the time increment. The solution of the value of the parameter $M = \dfrac{(\Delta x)^2}{\alpha^2 \Delta t}$ is important, as it governs the case with which we may proceed to effect the numerical solution.

Once the distance increment and the value of M are established, the time increment is fixed. The smaller values of Δx and Δt in the iteration process gives better accuracy. One important thing to note is that, if $M < 2$ the coefficients of equation (3.1.25) becomes negative and a condition is generated that the 2nd law of thermodynamics is violated.

The difference equation given above is useful for determining the internal temperature in the core as a function of space and time. Once the boundary conditions are specified, their finite difference approximations can be made and used to start the iteration process.

3.2   STEADY STATE NATURAL CONVECTION (THERMOSYPHON) COOLING

The most important aspect of the present work is concerned with the natural convection cooling (thermosyphoning) in the horizontal CANDU reactor core in the vicinity of each fuel rod following the transient, specifically in situations, when equilibrium natural circulation in the primary system cannot be established and auxilary recirculation pumps become inoperative.   A central question then becomes, whether thermosyphoning in the horizontal core plays dominant role in determining availability of adequate cooling following the accident, by directly extracting heat from the surface of the each fuel rod.

Problem Formulation :

A fuel bundle in a CANDU reactor core is a circular array of circular cylindrical fuel elements placed according to a specified array geometry.   The array is generally divided into triangular lattice cells as shown in Fig. 3.5, and coolant channel is characterized by its' hydraulic diameter, $D_e = \frac{4A_c}{P}$, where, A_c is the cross sectional area of the lattice cell and P is the wetted perimeter.   To formulate the problem, we consider individual fuel rods in the lattice cells, which are isolated from each other and try to develop an analysis for a single fuel rod. Without any ambiguity, the treatment can be extended for all the fuel rods assuming that these are all in the same thermal state. This seems reasonable because, the steady state heat flux distribution in the core has been considerably flattened compared to the initial sinusoidal distribution.   As a result, this gives

approximately a constant uniform heat flux throughout the core.

With the aid of the argument given by Lighthill [7], it is recalled that a boundary layer regime will be developed in the vicinity of each fuel rod because of the existence of considerably high heat flux inside the core. Calculation of steady state Grashoff number indicates that the boundary layer regime is laminar in nature. Since, the thickness of the thermal and hydrodynamic boundary layers are approximately equal in the fluids with Prandtl number close to unity, therefore, no separate boundary layers will be considered in the present treatment. The region near the fuel rod surface in the boundary layer regime, where velocity is found to be significant. The fluid beyond the boundary layer regime is assumed to be stationary, since the buoyant force due to gravity, causes the lighter fluid near the fluid to move and it becomes insignificant at a larger distance from the surface. The fluid velocity, therefore, be zero at the fuel rod surface, increase to a maximum value in the immediate neighbourhood, and then decreases asymptotically to zero at the outer edge of the boundary layer. The motion of the flow is restricted to a region close to the fuel rod surface. For the present treatment the following assumptions are made :

(1) Mutual influences of the fuel rods on their individual flow in the boundary layer regime is disregarded.

(2) The interaction of the boundary layer of a fuel rod with its' neighbouring one is also disregarded.

(3) The results derived for a single fuel rod is equally
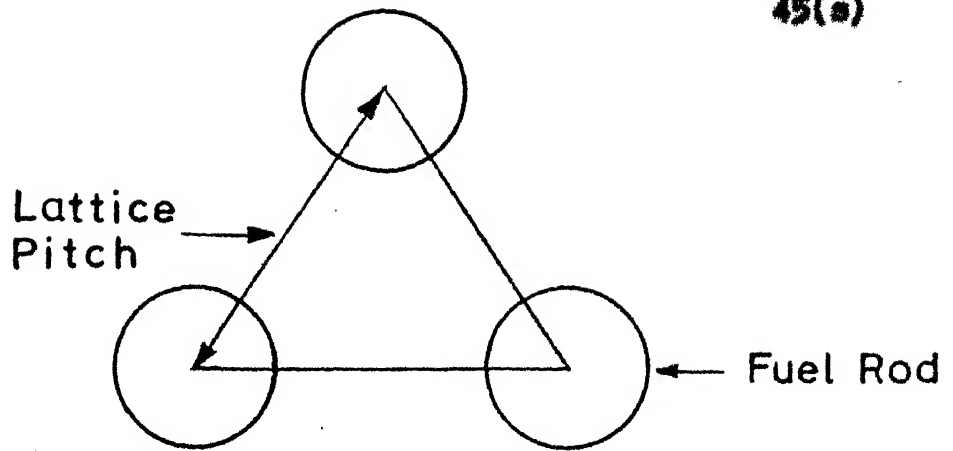
Fig.3·5 A Typical Triangular Lattice Cell
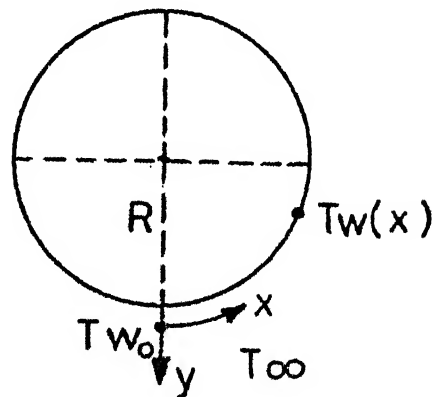
Fig. 3·6 Co-ordinate System
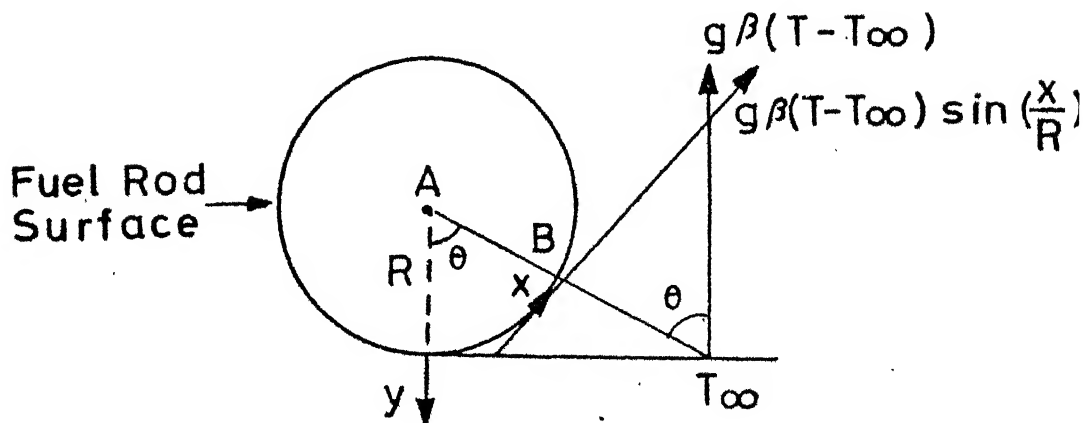
Fig. 3·7 Buoyant Force along the curved surface

applicable to all the rods inside a fuel bundle according to foregoing arguments.

Usually, Nusselt number and Grashoff number are based on the diameter of the cylinder, the significant length in all free convection problems with horizontal cylinders. A coordinate system suggested by Koh and Price [22] to treat axially symmetric horizontal objects in free convection is well suited for the present problem. This helps immensely to construct the buoyant force term. The coordinate system is shown in Fig. 3.6. It is located with the origin at the lower end of the cylinder. x coordinate varies across the fuel rod surface as shown in the figure and y axis is normal to the surface. Here T_{w_0}, $T_w(x)$ and T_∞ are temperatures at origin, at a point with coordinate x on the fuel rod surface and, ambient fluid (coolant) temperature respectively.

Isothermal and Non-isothermal surface conditions on the fuel rod :

As previously pointed out that, accumulation of volatile fission products in the fuel cladding gap and in other void regions in the cladding envelope is likely to affect the gap conductance and give rise to non-isothermal surface conditions on the fuel rod. In such situations, the temperature varies in an arbitrary manner with the coordinate x across the fuel rod surface. At x = 0, a stagnation point is formed. For the present situation, the surface temperature is assumed to vary in both linear and nonlinear manner as two separate cases. The surface

temperature is assumed to take the following forms:

For linear variation ;

$$(T_W - T_\infty) = (T_{W_0} - T_\infty)[1 + a(\frac{X}{R})] \qquad (3.2.1)$$

For nonlinear variation ;

$$(T_W - T_\infty) = (T_{W_0} - T_\infty)[1 + a(\frac{X}{R})^2] \qquad (3.2.2)$$

Where, a is an arbitrary constant and the surface temperature is always higher than the free stream (ambient) temperature T_∞ .

In isothermal surface condition, there is no variation of temperature occurs across the surface of the fuel rod, which implies that a = o. Hence, it is reasonable to suppose that the free convection heat transfer from a non-isothermal surface would be significantly different from that of an isothermal one.

Construction of the buoyant force term :

The density has been considered as only variable in forming the buoyancy force term. Variation of all other properties are neglected. The body force is assumed to be significant only in the vertically upward direction, then body force per unit volume in the vertical direction is $g\beta(T-T_\infty)$, where, g is the gravitational force vector and β is the thermal expansion coefficient. In order to construct the body force expression for a curved surface, a tangent is drawn at any point B on the fuel rod surface as shown in the Fig. 3.7. If R is the radius of the fuel rod and x is the coordinate of the point **B**, then the angle θ subtended at the centre of the circle

is equal to x/R. The direction of the buoyant force at the point B is along the direction of the tangent drawn at that point and its magnitude is $g\beta(T - T_\infty)\sin(x/R)$. Thus, generalizing the buoyancy force at any point on the curved surface of the fuel rod has a direction along the tangent drawn at that point and magnitude equal to $= g\beta(T - T_\infty)\sin(x/R)$, which is a function of the coordinate x.

Governing Differential Equations :

The two dimensional equations expressing conservation of mass, momentum, and energy for steady, laminar flow in a boundary layer on a horizontal cylindrical fuel element can be expressed, without any loss of generality in Cartesian coordinates as follows :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.2.3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T-T_\infty)\sin\left(\frac{x}{R}\right) \tag{3.2.4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\frac{\partial^2 T}{\partial y^2} \tag{3.2.5}$$

where, u and v are velocity components along x and y directions, respectively. ν is the kinematic viscosity and Pr is the Prandtl number.

Viscous dissipation and work against the gravity field have been omitted from the energy equation since they are negligibly small. The appropriate boundary conditions are :

$$y = 0 \qquad u = v = 0 \qquad T = T_w(x)$$
$$y \rightarrow \infty \qquad u \rightarrow 0 \qquad T \rightarrow T_\infty$$

First boundary conditions is the no slip condition on the surface and second one is the asymptotic boundary condition at the edge of the free convection boundary layer. Equation (3.2.3) can be automatically satisfied if a stream function is introduced such that

$$u = \frac{\partial \psi}{\partial y} \qquad v = - \frac{\partial \psi}{\partial x} \qquad (3.2.6)$$

The momentum and energy equations may be converted into dimensionless forms by introducing suitable similarity parameters as shown by Schimdt [23] and Valentine [24] for laminar free convection problems. The similarity parameters are given as:

$$\bar{x} = \frac{x}{R}$$

$$\eta = \left[\frac{g\beta (T_{w_o} - T_\infty) R^3}{\nu^2} \right]^{1/4} \frac{y}{R} = (Gr)^{1/4} \frac{y}{R}$$

$$M(\bar{x},\eta) = \left[\frac{\nu^2}{g\beta (T_{w_o} - T_\infty) R^3} \right]^{1/4} \frac{\psi}{\nu} = \frac{1}{(Gr)^{1/4}} \frac{\psi}{\nu}$$

$$(3.2.7)$$

$$\emptyset(\bar{x},\eta) = \frac{T - T_\infty}{T_{w_o} - T_\infty}$$

In the above transformations, the first two quantities, i.e. $\bar{x}$ and η represent dimensionless coordinates. $M(\bar{x},\eta)$ and $\emptyset(\bar{x},\eta)$ are dimensionless stream function and temperature respectively. The reasoning behind the selection of this form for the similarity parameters is rather involved and specific for particular type of problem. Using these transformation we deduce the following

$$u = \frac{\partial \Psi}{\partial y} = \frac{\nu}{R} (Gr)^{1/2} \frac{\partial M}{\partial \eta}$$

$$v = -\frac{\partial \Psi}{\partial x} = -\frac{(Gr)^{1/4} y}{R} \frac{\partial M}{\partial \bar{x}}$$

$$\frac{\partial u}{\partial y} = \frac{\nu}{R^2} Gr^{3/4} \frac{\partial^2 M}{\partial \eta^2}$$

$$\frac{\partial u}{\partial x} = \frac{\nu}{R^2} Gr^{1/2} \frac{\partial^2 M}{\partial \bar{x} \partial \eta}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\nu}{R^3} Gr \frac{\partial^3 M}{\partial \eta^3}$$

$$\frac{\partial T}{\partial x} = \frac{1}{R} (T_{w_0} - T_\infty) \frac{\partial \phi}{\partial \bar{x}}$$

$$\frac{\partial T}{\partial y} = (T_{w_0} - T_\infty) \frac{Gr^{1/4}}{R} \frac{\partial \phi}{\partial \eta}$$

$$\frac{\partial^2 T}{\partial y^2} = (T_{w_0} - T_\infty) \frac{Gr^{1/4}}{R^2} \frac{\partial^2 \phi}{\partial \eta^2}$$

Substituting the above results, the momentum and energy equations i.e. equation (3.2.4) and (3.2.5) may be rewritten in the following dimensionless form :

$$\frac{\partial M}{\partial \eta} \frac{\partial^2 M}{\partial \bar{x} \partial \eta} - \frac{\partial^2 M}{\partial \eta^2} \frac{\partial M}{\partial x} = \frac{\partial^3 M}{\partial \eta^3} + \phi \sin \bar{x} \qquad (3.2.8)$$

$$\frac{\partial M}{\partial \eta} \frac{\partial \phi}{\partial \bar{x}} - \frac{\partial M}{\partial \bar{x}} \frac{\partial \phi}{\partial \eta} = \frac{1}{Pr} \frac{\partial^2 \phi}{\partial \eta^2} \qquad (3.2.9)$$

In terms of the new variables, the boundary conditions are

$$\eta = o \qquad M = \frac{\partial M}{\partial \bar{x}} = \frac{\partial M}{\partial \eta} = 0 \qquad (3.2.10)$$

and, $\emptyset = 1$ (Isothermal case)

$\emptyset = (1 + a\bar{x})$ (Linear variation of temperature)

$\emptyset = (1 + a\bar{x}^2)$ (Non-linear variation of temperature)

$\eta \longrightarrow \infty$ $\qquad \dfrac{\partial M}{\partial \eta} \longrightarrow 0$ $\qquad \emptyset \longrightarrow 0$

Solution Procedure :

Equations (3.2.8) and (3.2.9) together with the boundary conditions can be solved by using Blasius-Howarth series approximation technique for the dimensionless stream function M and dimensionless temperature $\emptyset$ as discussed by Chiang and Kaye [25]. Thus, the series approximations for M and $\emptyset$ take the following forms for non-linear and linear variations of the fuel rod surface temperature. For non-linear case :

$$M(\bar{x},\eta) = \bar{x}\ F_o(\eta) + \bar{x}^3 F_1(\eta) + \ldots \qquad (3.2.11)$$

$$\emptyset(\bar{x},\eta) = G_o(\eta) + \bar{x}^2 G_1(\eta) + \ldots$$

and For linear case :

$$M(\bar{x},\eta) = \bar{x}\ F_o(\eta) + \bar{x}^2 F_1(\eta) + \ldots \qquad (3.2.12)$$

$$\emptyset(\bar{x},\eta) = G_o(\eta) + \bar{x}\ G_1(\eta) + \ldots$$

The first terms in each of these series expansion is the dominating one with respect to the other terms. Higher order terms are treated as perturbations to the first term. The great advantage lies with these series expansions, which convert the governing partial differential equation to a set of ordinary differential equations.

Sin $\bar{x}$ in equation (3.2.8) is approximated by the first two terms of its power series as follows :

$$\text{Sin } \bar{x} = \bar{x} - \frac{\bar{x}^3}{3!} + \dots \tag{3.2.13}$$

When equations (3.2.11) and sine series approximation (3.2.13) are substituted into equation (3.2.8) and (3.2.9) and terms arranged in ascending power of $\bar{x}$ gives.

$$\bar{x}[\frac{d^3F_o}{d\eta^3} + G_o - (\frac{dF_o}{d\eta})^2 + F_o \frac{d^2F_o}{d\eta^2}] + \bar{x}^3[\frac{d^3F_1}{d\eta^3} + G_1 - \frac{G_o}{6} - \frac{dF_1}{d\eta}\frac{dF_o}{d\eta}]$$

$$+ \bar{x}^5[3F_1 \frac{d^2F_1}{d\eta^2} - 3(\frac{dF_1}{d\eta})^2 - \frac{G_1}{6}] +[- 3\frac{dF_o}{d\eta}\frac{dF_1}{d\eta} + 3F_1\frac{d^2F_o}{d\eta^2} + F_o\frac{d^2F_1}{d\eta^2}]=0$$

and, $\hspace{8cm}$ (3.2.14)

$$(\frac{1}{Pr}\frac{d^2G_o}{d\eta^2} + F_o\frac{dG_o}{d\eta}) + \bar{x}^2(\frac{1}{Pr}\frac{d^2G_1}{d\eta^2} - 2G_1\frac{dF_o}{d\eta} + F_o\frac{dG_1}{d\eta} + 3F_1\frac{dG_o}{d\eta})$$

$$+ \bar{x}^4 (F_o\frac{dG_1}{d\eta} - 2\frac{dF_1}{d\eta}G_1) = 0 \tag{3.2.15}$$

Now, setting the coefficient of $\bar{x}^\alpha$ to zero for each non-negative integer $\alpha \leqslant 3$ and neglecting the coefficient of higher order terms in $\bar{x}$, yields the following sets of ordinary differential equation

$$\frac{d^3F_o}{d\eta^3} + F_o\frac{d^2F_o}{d\eta^2} - (\frac{dF_o}{d\eta})^2 + G_o = 0$$

$$\hspace{10cm} (3.2.16)$$

$$\frac{1}{Pr}\frac{d^2G_o}{d\eta^2} + F_o\frac{dG_o}{d\eta} = 0$$

$$\frac{d^3F_1}{d\eta^3} + F_o\frac{d^2F_1}{d\eta^2} - 4\frac{dF_o}{d\eta}\frac{dF_1}{d\eta} + 3\frac{d^2F_o}{d\eta^2}F_1 + G_1 - \frac{1}{6}G_o = 0$$

$$\hspace{10cm} (3.2.17)$$

$$\frac{1}{Pr} \frac{d^2 G_1}{d\eta^2} + F_o \frac{dG_1}{d\eta} - 2 \frac{dF_o}{d\eta} G_1 + 3 \frac{dG_o}{d\eta} F_1 = 0$$

The boundary conditions are

$$\eta = 0 \qquad F_o = \frac{dF_o}{d\eta} = 0 \qquad G_o = 1$$

$$F_1 = \frac{dF_1}{d\eta} = 0 \qquad G_1 = a \qquad (3.2.18)$$

$$\eta \longrightarrow \infty \qquad \frac{dF_o}{d\eta} \longrightarrow 0 \qquad G_o \longrightarrow 0$$

$$\frac{dF_1}{d\eta} \longrightarrow 0 \qquad G_1 \longrightarrow 0$$

Similarly, when equations (3.2.12) together with the sine series expansion (3.2.13) are substituted into equations (3.2.8) and (3.2.9) and terms arranged in ascending powers of $\bar{x}$, the setting to zero of the coefficients of $\bar{x}^{\alpha}$ for each non-negative integer $\alpha \leq 2$ yields the same sets of ordinary differential equations as given above.

An inspection of equations (3.2.16) to (3.2.18) reveals that the present problem involves two parameters, Pr and a. However, for isothermal surface condition 'a' becomes zero. For non-isothermal conditions, for each combination of Pr and a it would be necessary to solve two pairs of simultaneous equations. The Prandtl number is considered to be a constant. However, a can have different values as it is taken to be an arbitrary constant. Hence, it may be desired to solve equations (3.2.16) to (3.2.18) for different values of a. Consequently, refering to Struble [26], the following transformations are used to transform

the original equations and boundary conditions to new sets of differential equations and boundary conditions which are free of the parameter a.

$$F_0(\eta) = X_1(\eta) \qquad\qquad G_0(\eta) = Y_1(\eta)$$

$$F_1(\eta) = aX_2(\eta) + X_3(\eta) \qquad G_1(\eta) = aY_2(\eta) + Y_3(\eta)$$

(3.2.19)

Substituting these transformations into equations (3.2.16) to (3.2.18), and with little algebra yields the new differential equations together with their boundary conditions.

$$\frac{d^3X_1}{d\eta^3} + X_1 \frac{d^2X_1}{d\eta^2} - \left(\frac{dX_1}{d\eta}\right)^2 + Y_1 = 0$$

(3.2.20)

$$\frac{d^2Y_1}{d\eta^2} + Pr\, X_1 \frac{dY_1}{d\eta} = 0$$

$$\eta = 0 \qquad X_1 = 0 \qquad \frac{dX_1}{d\eta} = 0 \qquad Y_1 = 1$$

$$\eta \longrightarrow \infty \qquad \frac{dX_1}{d\eta} \longrightarrow 0 \qquad\qquad Y_1 \longrightarrow 0$$

$$\frac{d^3X_2}{d\eta^3} + X_1 \frac{d^2X_2}{d\eta^2} - 4 \frac{dX_1}{d\eta} \frac{dX_2}{d\eta} + 3 \frac{d^2X_1}{d\eta^2} X_2 - Y_2 = 0$$

(3.2.21)

$$\frac{d^2Y_2}{d\eta^2} + Pr\left[X_1 \frac{dY_2}{d\eta} - 2 \frac{dX_1}{d\eta} Y_2 + 3X_2 \frac{dY_1}{d\eta}\right] = 0$$

$$\eta = 0 \qquad X_2 = 0 \qquad \frac{dX_2}{d\eta} = 0 \qquad Y_2 = 1$$

$$\eta \longrightarrow \infty \qquad \frac{dX_2}{d\eta} \longrightarrow 0 \qquad\qquad Y_2 \longrightarrow 0$$

$$\frac{d^3X_3}{d\eta^3} + X_1 \frac{d^2X_3}{d\eta^2} - 4 \frac{dX_1}{d\eta} \frac{dX_3}{d\eta} + 3X_3 \frac{d^2X_1}{d\eta^2} + Y_3 - \frac{1}{6} Y_1 = 0$$

$$\frac{d^2Y_3}{d\eta^2} + Pr[X_1 \frac{dY_3}{d\eta} - 2 \frac{dX_1}{d\eta} Y_3 + 3X_3 \frac{dY_1}{d\eta}] = 0$$

$$(3.2.22)$$

$$\eta = 0 \qquad X_3 = 0 \qquad \frac{dX_3}{d\eta} = 0 \qquad Y_3 = 0$$

$$\eta \longrightarrow \infty \qquad \frac{dX_3}{d\eta} \longrightarrow 0 \qquad Y_3 \longrightarrow 0$$

The equations (3.2.20) to (3.2.22) are free of the parameter a and hence solutions of these equations are applicable for different values of a.

It is noted that equations (3.2.20) are non-linear, while remainder of the equations are linear (assuming that solutions of equations (3.2.20) are known). All of these three sets of equations are coupled third order ordinary differential equations with asymptotic boundary conditions. Inspection of equations (3.2.20) reveals that these are very much similar to the boundary value problem for the free convection flow about a vertical plate refering to Ostrach [27]. Equations (3.2.21) and (3.2.22) involve four dependent variables each, and X_1 and Y_1 are two of these four variables. These asymptotic boundary value problems are virtually impossible to solve by an analytical method. Extensive numerical analysis of these equations are required to solve them fairly accurately by a reasonably good numerical technique. The mathematical apparatus required for the numerical solution of these equations will be introduced in

succeeding chapter.

Velocity, Temperature and Heat transfer :

Substituting the transformations given by equations (3.2.19) into equations (3.2.11) and (3.2.12), the dimensionless stream functions and temperature assumes the following forms respectively for non-linear and linear variation of surface temperature.

$$M(\bar{x},\eta) = \bar{x}\, X_1(\eta) + \bar{x}^3\,(aX_2(\eta) + X_3(\eta))$$
$$\emptyset(\bar{x},\eta) = Y_1(\eta) + \bar{x}^2\,(aY_2(\eta) + Y_3(\eta))$$

$$(3.2.23)$$

$$M(\bar{x},\eta) = \bar{x}\, X_1(\eta) + \bar{x}^2\,(aX_2(\eta) + X_3(\eta))$$
$$\emptyset(\bar{x},\eta) = Y_1(\eta) + \bar{x}\,(aY_2(\eta) + Y_3(\eta))$$

$$(3.2.24)$$

From the expression for similarity parameter related to dimensionless stream function M, given in equation (3.2.7), we have

$$\psi = M\,(Gr)^{1/4}\,\nu, \qquad \text{which gives,}$$

$$u = \frac{\partial \psi}{\partial y} = \frac{(Gr)^{1/2}\nu}{R}\,\frac{\partial M(\bar{x},\eta)}{\partial \eta}$$

Substituting for $\frac{\partial M}{\partial \eta}$ from the equations (3.2.23) and (3.2.24) into the above equation, the expression for velocity distribution in the boundary layer follows for both non-linear and linear cases of thermal condition on the surface :

$$u = \frac{(Gr)^{1/2}\nu}{R}\,[\bar{x}\,\frac{dX_1}{d\eta} + \bar{x}^3\,(a\,\frac{dX_2}{d\eta} + \frac{dX_3}{d\eta})]$$

$$u = \frac{(Gr)^{1/2}\nu}{R}\,[\bar{x}\,\frac{dX_1}{d\eta} + \bar{x}^2\,(a\,\frac{dX_2}{d\eta} + \frac{dX_3}{d\eta})]$$

$$(3.2.25)$$

Now, equating Fourier law of heat flow on the surface in terms of $\emptyset$ and heat flux, q/A given as,

$$q/A = - K(T_{w_o} - T_\infty) \frac{Gr^{1/4}}{R} \frac{\partial \emptyset}{\partial \eta}\Big|_{\eta=0}$$

and Newton's law of cooling for the heat converted away from the surface given as,

$$q/A = h (T_{w_o} - T_\infty)$$

One finds the following equation,

$$h = - K \frac{Gr^{1/4}}{R} \frac{\partial \emptyset}{\partial \eta}\Big|_{\eta=0}$$

The dimensionless heat transfer i.e., Nusselt number may be given as,

$$Nu_D = \frac{hR}{K} = -(Gr)^{1/4} \frac{\partial \emptyset}{\partial \eta}\Big|_{\eta=0} \qquad (3.2.26)$$

where, Nu_D based upon the radius (i.e. diameter) of the fuel rod. Substituting for $\frac{\partial \emptyset}{\partial \eta}\Big|_{\eta=0}$ from æequations (3.2.23) and (3.2.24) into the above equation, the expression for Nusselt number follows for both non-linear and linear cases of thermal condition on the surface

$$Nu_D = - Gr^{1/4} [\frac{dY_1(0)}{d\eta} + \bar{x}^2(a \frac{dY_2(0)}{d\eta} + \frac{dY_3(0)}{d\eta})]$$

$$\qquad (3.2.27)$$

$$Nu_D = - Gr^{1/4} [\frac{dY_1(0)}{d\eta} + \bar{x} (a \frac{dY_2(0)}{d\eta} + \frac{dY_3(0)}{d\eta})]$$

The equations representing, velocity and Nusselt number for non-linear and linear cases are similar except for the power of $\bar{x}$ involved in those expressions. The velocity distribution in the

boundary layer accounts for the thermosyphoning effects, in the form of eddies in the vicinity of each fuel rod. In order to compute the velocity and Nusselt number, the derivatives in the corresponding expressions must be evaluated which are functions of η, the measure of the dimensionless distance from the surface of the fuel rod.

CHAPTER IV

NUMERICAL ANALYSIS OF THE SYSTEMS
OF BOUNDARY LAYER DIFFERENTIAL EQUATIONS

4.1 INTRODUCTION

The numerical integration of the ordinary differential equations, obtained from Blasius-Hawarth series with appropriate transformations, involves the satisfaction of asymptotic boundary conditions. That is, some boundary conditions are specified at the initial point or wall and other are specified as limits that must be approached at relatively large values of the independent variable, corresponding to the edge of the boundary layer. In order to integrate the differential equations numerically from the wall to the edge of the boundary layer, it is necessary to specify as many additional conditions at the wall as there are conditions to be satisfied at the edge of the boundary layer. All methods that have been used to find the proper initial conditions rely on the fact that, for relatively large values of the independently variable, the integrals of the differential equations depend on the initial conditions. Among the broad categories of numerical methods, two widely applicable techniques are namely :

(i) Initial value method, Fox [28].

(ii) Quasilinearization method, Radbill [29].

Initial value method, that has been used to find proper initial conditions, is that of obtaining integrals of the differential equations with guessed initial conditions, that is an attempt is made to integrate the differential equations to the edge of the boundary layer. If this can be accomplished, corrections are made for the initial guesses by suitable iterative method and the process is repeated untill desired accuracy is achieved. In order to carry out these iterations, additional differential equations have to be integrated. The additional differential equations are obtained by differentiating the terms of the original differential equations with respect to initial conditions. The integrals of these additional differential equations, referred to as perturbation equations, give the rate of change of the integrals of the original differential equations with respect to the initial conditions.

Quasilinearization is similar in principle to the initial value method, except that the original differential equations, if non-linear, are linearized. The resulting linear differential equations for the current approximation are inhomogeneous, since they also contain the previous approximation as members. In application, the two methods are different in that, in the quasilinearization method, the complementary solutions are used to obtain solutions to the inhomogeneous linear differential equations, whereas in initial value method, the integrals of the perturbation equations are used to adjust the starting values at the initial point.

A disadvantage associated with quasilinearization method is that, the solution of the inhomogeneous equations obtained by combining complementary solution and a particular integral is usually not well determined except near the origin or initial point as pointed out by Hartree [30]. Thus initial value method has an edge over the latter, as solutions of perturbation equations can be used to determine the proper initial conditions closely. Since, boundary conditions specified as a limit that must be approached at higher values of the independent variable, the integration should be stopped at a value of the independent variable, so that various functions are approaching their asymptotic and physically acceptable values. This value of the independent variable may be called approximately the edge of the boundary layer. This value of independent variable is obtained by suitable guess and trial effort.

In an effort to adapt the initial value method to the solution of differential equations with asymptotic boundary condition, an iterative method developed by Nachtsheim and Swigert [31] to correct the initial conditions, is applied. This method of solution is capable of satisfying boundary layer correctly. This is accomplished by choosing the additional initial conditions so that the mean square error between the computed variable and asymptotic variable is minimized. This method of solution is based on least square error criterion and the edge of the boundary layer is approached in steps.

4.2 DESCRIPTION OF THE METHOD

A description and application of the initial value method can best be given, refering to any one of the set of differential equations as an example. However, the general iterative formulas derived by using first set of non-linear equation (3.2.20) are equally applicable to all the sets of equations.

Using 'primes', to denote differentiation with respect to independent variable η, the measure of the dimensionless distance from the wall, we have:

$$\frac{dX_1}{d\eta} = X_1' \ , \quad \frac{d^2X_1}{d\eta^2} = X_1'' \ , \quad \frac{d^3X_1}{d\eta^3} = X_1''' \ .$$

Now, set of equations (3.2.20) can be rewritten as

$$X_1''' = -(X_1 X_1'' - X_1'^2 + Y_1)$$

$$Y_1' = - Pr \ X_1 \ Y_1' \qquad\qquad (4.2.1)$$

with boundary conditions:

$$\eta = 0, \quad X_1(\eta) = X_1'(\eta) = 0, \quad Y_1(\eta) = 1$$

$$\eta \rightarrow \infty, \quad Y_1(\eta) = X_1'(\eta) \longrightarrow 0$$

The dependent variables, X_1 and Y_1 related to dimensionless stream function and temperature respectively.

In practice, the asymptotic boundary conditions is replaced by conditions that $X_1' = 0$ and $Y_1 = 0$, to a sufficient degree of accuracy at $\eta = \eta_{edge}$, where η_{edge} is the value of

the independent variable. The boundary value problem of this coupled differential equation is equivalent to the problem of finding values of $X_1''(0)$ and $Y_1'(0)$ for which the boundary conditions at the edge of the boundary layer is satisfied, i.e. it is desired to find solutions of equations

$$X_1'{}_{edge} \, [X_1''(0), \, Y_1'(0)] \; = \; 0 \qquad\qquad (4.2.2)$$

$$Y_1{}_{edge} \, [X_1''(0), \, Y_1'(0)] \; = \; 0 \qquad\qquad (4.2.3)$$

where, $X_1'{}_{edge} \equiv X_1'(\eta_{edge})$ and $Y_1{}_{edge} \equiv Y_1(\eta_{edge})$. Since there are two asymptotic boundary conditions to satisfy, two additional initial conditions $X_1''(0)$ and $Y_1'(0)$ at the wall will have to be adjusted, that is, values of these are sought that will satisfy simultaneously the above equations. The functions $X_1'{}_{edge}[X_1''(0), \, Y_1'(0)]$ and $Y_1{}_{edge}[X_1''(0), \, Y_1'(0)]$ in this problem are not, in general, explicit; those will be expressed as functions of $X_1''(0)$ and $Y_1'(0)$ through integration of equation (4.2.1).

Using notations[*] $x = X_1''(0)$ and $y = Y_1'(0)$, we observed that, small changes $\triangle x$ and $\triangle y$ in the initial conditions change X_1' and Y_1 by the amounts $\frac{\partial X_1'}{\partial x}\, \triangle x + \frac{\partial X_1'}{\partial y}\, \triangle y$ and $\frac{\partial Y_1}{\partial x}\triangle x + \frac{\partial Y_1}{\partial y}\, \triangle y$ respectively. The necessary correction to the first approximation comes from the solution of the linear equations

$$X_1' + \frac{\partial X_1'}{\partial x}\, \triangle x + \frac{\partial X_1'}{\partial y}\, \triangle y \; = \; 0 \qquad\qquad (4.2.4)$$

$$Y_1 + \frac{\partial Y_1}{\partial x}\, \triangle x + \frac{\partial Y_1}{\partial y}\, \triangle y \; = \; 0 \qquad\qquad (4.2.5)$$

at $\eta = \eta_{edge}$.

[*] The notations here should not be confused with the coordinates

The initial value method again demands that all higher
derivatives of the independent variables vanish at the outer
boundary i.e. at $\eta = \eta_{edge}$. Hence, satisfaction of both pair
of boundary conditions i.e. $X_1' = 0$, $X_1'' = 0$ and $Y_1 = 0$, $Y_1' = 0$
at $\eta = \eta_{edge}$ ensures that X_1' and Y_1 approach zero asymptotically
as η approaches infinity. Thus, in order to satisfy asymptotic
conditions at a finite value of η, both pair of above conditions
must be imposed. Hence, in order to satisfy, asymptotic
boundary conditions, the preceeding equations (4.2.2) and
(4.2.3) must be supplemented by

$$X_1'' {}_{edge} [X_1'' (0), Y_1' (0)] = 0 \qquad (4.2.6)$$

$$Y_1'' {}_{edge} [X_1'' (0), Y_1' (0)] = 0 \qquad (4.2.7)$$

Again for small changes $\triangle x$ and $\triangle y$ in the initial conditions,
the necessary correction to the first approximation, as
previously given, comes from the solution of linear equations

$$X_1'' + \frac{\partial X_1''}{\partial x} \triangle x + \frac{\partial X_1''}{\partial y} \triangle y = 0 \qquad (4.2,8)$$

$$Y_1' + \frac{\partial Y_1'}{\partial x} \triangle x + \frac{\partial Y_1'}{\partial y} \triangle y = 0 \qquad (4.2.9)$$

at $\eta = \eta_{edge}$.

The solution of both pairs of equations i.e. (4.2.4),
(4.2.5) and (4.2.8), (4.2.9) can be performed, provided that
the partial derivatives involves in those equations with respect
to x and y can be evaluated at $\eta = \eta_{edge}$. The partial deriva-
tives can be evaluated by forming and integrating perturbation

equations for the functions X_1', X_1'' and Y_1, Y_1'. These equations are obtained by differentiating the terms in the original equations (4.2.1) approximately with respect to x and y respectively.

The perturbation differential equation for x derivatives are :

$$\frac{\partial X_1'''}{\partial x} = -\left(\frac{\partial X_1}{\partial x} X_1'' + X_1 \frac{\partial X_1''}{\partial x}\right) + 2X_1' \frac{\partial X_1'}{\partial x} - \frac{\partial Y_1}{\partial x}$$

$$(4.2.10)$$

$$\frac{\partial Y_1''}{\partial x} = -Pr\left(\frac{\partial X_1}{\partial x} Y_1' + X_1 \frac{\partial Y_1}{\partial x}\right)$$

with the initial conditions

$$\eta = 0: \quad \frac{\partial X_1}{\partial x} = \frac{\partial X_1'}{\partial x} = \frac{\partial Y_1}{\partial x} = \frac{\partial Y_1'}{\partial x} = 0, \quad \frac{\partial X_1''}{\partial x} = 1$$

The perturbation differential equations for the y derivatives are

$$\frac{\partial X_1'''}{\partial y} = -\left(\frac{\partial X_1}{\partial y} X_1'' + X_1 \frac{\partial X_1''}{\partial y}\right) + 2X_1' \frac{\partial X_1'}{\partial y} - \frac{\partial Y_1}{\partial y}$$

and

$$(4.2.11)$$

$$\frac{\partial Y_1'''}{\partial y} = -Pr\left(\frac{\partial X_1}{\partial y} Y_1' + X_1 \frac{\partial Y_1}{\partial y}\right)$$

with initial conditions

$$\eta = 0: \quad \frac{\partial X_1}{\partial y} = \frac{\partial X_1'}{\partial y} = \frac{\partial X_1''}{\partial y} = \frac{\partial Y_1}{\partial y} = 0, \quad \frac{\partial Y_1'}{\partial y} = 1.$$

Although, the equations for the y derivatives are the same as the equations for the x derivatives, but the integrals . of these equations will differ since the initial conditions are different. It is also noted that there are as many system of

perturbation equations to integrate as there are asymptotic boundary conditions to meet.

Now, corrections to the initial conditions $\triangle x$ and $\triangle y$ so chosen that these four equations (4.2.2), (4.2.3), (4.2.8) and (4.2.9) are satisfied at $\eta = \eta_{edge}$. This is, in general, impossible since there are four equations and only two adjustable parameters. However, a satisfactory solution that is consistent with the idea that the boundary conditions can't be satisfied exactly at a finite value of η, is to seek least square solutions of those four equations.

4.3 NACHTSHEIM-SWIGERT ITERATION SCHEME

The method developed by Nachtsheim and Swigert [31] can be applied to these four equations to solve for $\triangle x$ and $\triangle y$ basing on least square error criteria. According to the method, $\triangle x$ and $\triangle y$ should be found such that, the sum of the squares of the error in these four equations must be minimum. Hence, considerang discrepancies δ_1, δ_2, δ_3 and δ_4 in these equations respectively (deviations from their asymptotic values), where,

$$X_1' + \frac{\partial X_1'}{\partial x} \triangle x + \frac{\partial X_1'}{\partial y} \triangle y = \delta_1$$

$$Y_1 + \frac{\partial Y_1}{\partial x} \triangle x + \frac{\partial Y_1}{\partial y} \triangle y = \delta_2$$

$$X_1'' + \frac{\partial X_1''}{\partial x} \triangle x + \frac{\partial X_1''}{\partial y} \triangle y = \delta_3 \qquad (4.3.1)$$

$$Y_1' + \frac{\partial Y_1'}{\partial x} \triangle x + \frac{\partial Y_1'}{\partial y} \triangle y = \delta_4$$

It is equivalent of saying that, the original equations (4.2.2)

and (4.2.3) along with the supplemented equations (4.2.6) and (4.2.7) are associated with these descripancies δ_1, δ_2, δ_3 and δ_4 respectively. We have :

$$X'_{1\ edge}\ [X''_1\ (0),\ Y'_1\ (0)]\ =\ \delta_1$$

$$Y_{1\ edge}\ [X''_1\ (0),\ Y'_1\ (0)]\ =\ \delta_2$$

$$X''_{1\ edge}\ [X''_1\ (0),\ Y'_1\ (0)]\ =\ \delta_3$$

$$Y'_{1\ edge}\ [X''_1\ (0),\ Y'_1\ (0)]\ =\ \delta_4$$

(4.3.2)

Solution of set of equations (4.3.1) in least square sense requires determination of mininum values of the error,

$$E\ =\ \delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2$$

So differentiating E with respect to x and y respectively and equating to zero i.e. we evaluate: For x derivatives,

$$\frac{\partial E}{\partial x}\ =\ \frac{\partial}{\partial x}\ (\delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2)\ =\ 0.$$ Substituting for δ_1^2, δ_2^2, δ_3^2 and δ_4^2 from the set of equations (4.3.1), we have after rearranging terms:

$$[(\frac{\partial X'_1}{\partial x})^2 + (\frac{\partial Y_1}{\partial x})^2 + (\frac{\partial X''_1}{\partial x})^2 + (\frac{\partial Y'_1}{\partial x})^2]\ \Delta x + [\frac{\partial X'_1}{\partial x}\frac{\partial X'_1}{\partial y} + \frac{\partial Y_1}{\partial x}\frac{\partial Y_1}{\partial y}$$

$$+ \frac{\partial X''_1}{\partial x}\frac{\partial X''_1}{\partial y} + \frac{\partial Y'_1}{\partial x}\frac{\partial Y'_1}{\partial y}]\ \Delta y\ =\ -\ (X'_1 \frac{\partial X'_1}{\partial x} + Y_1 \frac{\partial Y_1}{\partial y} + X''_1 \frac{\partial X''_1}{\partial x} + Y'_1 \frac{\partial Y'_1}{\partial x})$$

(4.3.3)

Similarly, for y derivative, equating $\frac{\partial E}{\partial y}$ to zero, and rearranging terms,

$$[\frac{\partial X_1'}{\partial x}\frac{\partial X_1'}{\partial y} + \frac{\partial Y_1}{\partial x}\frac{\partial Y_1}{\partial y} + \frac{\partial X_1''}{\partial x}\frac{\partial X_1''}{\partial y} + \frac{\partial Y_1''}{\partial x}\frac{\partial Y_1'}{\partial y}]\triangle x + [(\frac{\partial X_1'}{\partial y})^2 + (\frac{\partial Y_1}{\partial y})^2$$

$$+ (\frac{\partial X_1''}{\partial y})^2 + (\frac{\partial Y_1'}{\partial y})^2]\triangle y = -(X_1'\frac{\partial X_1'}{\partial y} + Y_1\frac{\partial Y_1}{\partial y} + X_1''\frac{\partial X_1''}{\partial y} + Y_1'\frac{\partial Y_1'}{\partial y})$$

$$(4.3.4)$$

Equations (4.3.3) and (4.3.4) can be written as a system of linear equations in two unknowns in a simplified form.

$$A_{xx}\triangle x + A_{xy}\triangle y = B_1$$

$$A_{xy}\triangle x + A_{yy}\triangle y = B_2$$

$$(4.3.5)$$

and in matrix notation,

$$\begin{pmatrix} A_{xx} & A_{xy} \\ A_{xy} & A_{yy} \end{pmatrix}\begin{pmatrix} \triangle x \\ \triangle y \end{pmatrix} = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

where,

$$A_{xx} = (\frac{\partial X_1'}{\partial x})^2 + (\frac{\partial Y_1}{\partial x})^2 + (\frac{\partial X_1''}{\partial x})^2 + (\frac{\partial Y_1'}{\partial x})^2$$

$$A_{xy} = (\frac{\partial X_1'}{\partial x}\frac{\partial X_1'}{\partial y} + \frac{\partial Y_1}{\partial x}\frac{\partial Y_1}{\partial y} + \frac{\partial X_1''}{\partial x}\frac{\partial X_1''}{\partial y} + \frac{\partial Y_1'}{\partial x}\frac{\partial Y_1'}{\partial y})$$

$$A_{yy} = (\frac{\partial X_1'}{\partial y})^2 + (\frac{\partial Y_1}{\partial y})^2 + (\frac{\partial X_1''}{\partial y})^2 + (\frac{\partial Y_1'}{\partial y})^2$$

$$B_1 = -(X_1'\frac{\partial X_1'}{\partial x} + Y_1\frac{\partial Y_1}{\partial x} + X_1''\frac{\partial X_1''}{\partial x} + Y_1'\frac{\partial Y_1'}{\partial x})$$

$$B_2 = -(X_1'\frac{\partial X_1'}{\partial y} + Y_1\frac{\partial Y_1}{\partial y} + X_1''\frac{\partial X_1''}{\partial y} + Y_1'\frac{\partial Y_1'}{\partial y})$$

Solving equations (4.3.5) for the two unknowns $\triangle x$ and $\triangle y$, we have,

$$\Delta x = \frac{B_1 A_{yy} - B_2 A_{xy}}{A_{xx} A_{yy} - A_{xy}^2} \qquad (4.3.6)$$

and

$$\Delta y = \frac{A_{xx} B_2 - B_1 A_{xy}}{A_{xx} A_{yy} - A_{xy}^2} \qquad (4.3.7)$$

This calculation yields values of Δx and Δy corresponding minimum of the sum $\delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2$. The magnitude of error E, evaluated for a given range of independent variable approached in steps, gives an indication how unsatisfactory the asymptotic boundary conditions are. The error term can also be written as

$$E = X_1'^2 + Y_1^2 + X_1''^2 + Y_1'^2 \qquad (4.3.8)$$

The minimum of E with respect to x and y, say E_{min}, corresponds to no deviations in the required initial conditions, that permits the asymptotic boundary conditions to be approximately satisfied at a finite value of the independent variable. The edge of the boundary layer, taken to be some guessed value of η consistent with our assumptions, for which E_{min} is less than some preassigned small value for this range of integration, gives the desired solutions.

These modifications to the initial value method come from the corrections to the initial conditions given by equations (4.3.6) and (4.3.7) in a least square sense. These are generalized correction formulas and can be applied to any system of coupled third order differential equations, if the subscript '1' associated with X and Y is ignored. However, the main difference

lies with the evaluation of corresponding partial derivatives involved in correction equations, which would be different for different set of equations and also correspondingly their perturbation equations.

Application to systems of equations (3.2.21) and (3.2.22):

In primed notations as stated before, these equations can be rewritten as:

$$X_2''' = -(X_1 X_2'' - 4X_1' X_2' + 3X_1'' X_2 - Y_2)$$

$$Y_2'' = -Pr\ (X_1 Y_2' - 2X_1' Y_2 + 3Y_1' X_2)$$

$$(4.3.9)$$

with boundary conditions:

$$\eta = 0, \quad X_2 = X_2' = 0, \quad Y_2 = 1$$

$$\eta \to \infty, \quad X_2' \to 0 \quad Y_2 \to 0$$

and,

$$X_3''' = -(X_1 X_3'' - 4X_1' X_3' + 3X_3 X_1'' + Y_3 - \tfrac{1}{6}Y_1)$$

$$Y_3'' = -Pr\ (X_1 Y_3' - 2X_1' Y_3 + 3X_3 Y_1')$$

$$(4.3.10)$$

with boundary conditions:

$$\eta = 0, \quad X_3 = X_3' = 0, \quad Y_3 = 0$$

$$\eta \to \infty, \quad X_3' \to 0 \quad Y_3 \to 0$$

In the above equations, dependent variables each. These equations can only be solved by knowing the solution, namely, the values of corrected initial conditions $X_1''(0)$ and $Y_1(0)$ of

equation (2.4.1). Method of solution of these equations is
exactly same as given in the foregoing analysis, once the values
of the initial conditions are evaluated. The only difference
lies with the evaluation of partial derivatives involved impli-
citly in the expressions (4.3.6) and (4.3.7), which would be
different for different system of coupled equations depending
upon the integrals of corresponding perturbation equations.

The perturbation differential equations for the
coupled equations (4.3.9) are obtained by differentiating
original equations with respect to the initial conditions x and
y, respectively. For x derivative,

$$\frac{\partial X_2'''}{\partial x} = - \left(\frac{\partial X_1}{\partial x} X_2'' + X_1 \frac{\partial X_2''}{\partial x} - 4X_1' \frac{\partial X_2'}{\partial x} - 4 \frac{\partial X_1'}{\partial x} X_2' \right.$$

$$\left. + 3 \frac{\partial X_1''}{\partial x} X_2 + 3X_1'' \frac{\partial X_2}{\partial x} - \frac{\partial Y_2}{\partial x} \right)$$

$$(4.3.11)$$

$$\frac{\partial Y_2''}{\partial x} = - Pr \left(X_1 \frac{\partial Y_2'}{\partial x} + Y_2' \frac{\partial X_1}{\partial x} - 2X_1' \frac{\partial Y_2}{\partial x} \right.$$

$$\left. - 2 \frac{\partial X_1'}{\partial x} Y_2 + 3X_2 \frac{\partial Y_1'}{\partial x} + 3 \frac{\partial X_2}{\partial x} Y_1' \right)$$

With initial conditions:

$$\eta = 0: \quad \frac{\partial X_2}{\partial x} = \frac{\partial X_2'}{\partial x} = \frac{\partial Y_2}{\partial x} = \frac{\partial Y_2'}{\partial x} = 0, \quad \frac{\partial X_2''}{\partial x} = 1$$

$$\eta = 0: \quad \frac{\partial X_1}{\partial x} = \frac{\partial X_1'}{\partial x} = \frac{\partial Y_1}{\partial x} = \frac{\partial Y_1'}{\partial x} = 0, \quad \frac{\partial X_1''}{\partial x} = 1$$

For y derivatives,

$$\frac{\partial X_2'''}{\partial y} = - \left(\frac{\partial X_1}{\partial y} X_2'' + X_1 \frac{\partial X_2''}{\partial y} - 4X_1' \frac{\partial X_2'}{\partial y} - 4 \frac{\partial X_1'}{\partial x} X_2' \right.$$

$$\left. + 3 \frac{\partial X_1''}{\partial y} X_2 + 3X_1'' \frac{\partial X_2}{\partial y} - \frac{\partial Y_2}{\partial x} \right)$$

$$(4.3.12)$$

$$\frac{\partial Y_2''}{\partial y} = - Pr \left(X_1 \frac{\partial Y_2'}{\partial y} + Y_2' \frac{\partial X_1}{\partial y} + 3X_2 \frac{\partial Y_1'}{\partial x} + 3 \frac{\partial X_2}{\partial x} Y_1' \right)$$

With initial conditions:

$$\eta = 0: \quad \frac{\partial X_2}{\partial y} = \frac{\partial X_2'}{\partial y} = \frac{\partial X_2''}{\partial y} = \frac{\partial Y_2}{\partial y} = 0, \quad \frac{\partial Y_2'}{\partial y} = 1$$

$$\eta = 0: \quad \frac{\partial X_1}{\partial y} = \frac{\partial X_1'}{\partial y} = \frac{\partial X_1''}{\partial y} = \frac{\partial Y_1}{\partial y} = 0, \quad \frac{\partial Y_1'}{\partial y} = 1$$

Similarly, the perturbation differential equations for the system of coupled differential equations (4.3.10) are as follows:

For x derivative,

$$\frac{\partial X_3'''}{\partial x} = - \left(\frac{\partial X_1}{\partial x} X_3'' + X_1 \frac{\partial X_3''}{\partial x} - 4X_1' \frac{\partial X_3'}{\partial x} - 4 \frac{\partial X_1'}{\partial x} X_3' \right.$$

$$\left. + 3 \frac{\partial X_1''}{\partial x} X_3 + 3X_1'' \frac{\partial X_3}{\partial x} + \frac{\partial X_3}{\partial x} - \frac{1}{6} \frac{\partial Y_1}{\partial x} \right)$$

$$(4.3.13)$$

$$\frac{\partial Y_3''}{\partial x} = -Pr \left(X_1 \frac{\partial Y_3'}{\partial x} + Y_3' \frac{\partial X_1}{\partial x} - 2 \frac{\partial X_1'}{\partial x} Y_3 - 2X_1' \frac{\partial Y_3}{\partial x} \right.$$

$$\left. + 3X_3 \frac{\partial Y_1'}{\partial x} + 3 \frac{\partial X_3}{\partial x} Y_1 \right)$$

With initial conditions:

$\eta=0$: $\quad \dfrac{\partial X_3}{\partial x} = \dfrac{\partial X_3'}{\partial x} = \dfrac{\partial Y_3}{\partial x} = \dfrac{\partial Y_3'}{\partial x} = 0, \qquad \dfrac{\partial X_3''}{\partial x} = 1$

$\eta=0$: $\quad \dfrac{\partial X_1}{\partial x} = \dfrac{\partial X_1'}{\partial x} = \dfrac{\partial Y_1}{\partial x} = \dfrac{\partial Y_1'}{\partial x} = 0, \qquad \dfrac{\partial X_1''}{\partial x} = 1$

For y derivatives:

$$\dfrac{\partial X_3'''}{\partial y} = -\left(\dfrac{\partial X_1}{\partial y} X_3'' + X_1 \dfrac{\partial X_3''}{\partial y} - 4X_1' \dfrac{\partial X_3'}{\partial y} - 4 \dfrac{\partial X_1'}{\partial y} X_3' \right.$$

$$\left. + 3 \dfrac{\partial X_1''}{\partial y} X_3 + 3X_1'' \dfrac{\partial X_3}{\partial y} + \dfrac{\partial Y_3}{\partial y} - \dfrac{1}{6} \dfrac{\partial Y_1}{\partial y} \right)$$

$$(4.3.14)$$

$$\dfrac{\partial Y_3''}{\partial y} = -\mathrm{Pr}\left(X_1 \dfrac{\partial Y_3'}{\partial y} + Y_3' \dfrac{\partial X_1}{\partial y} - 2 \dfrac{\partial X_1'}{\partial y} Y_3 - 2X_1' \dfrac{\partial Y_3}{\partial y} \right.$$

$$\left. + 3X_3 \dfrac{\partial Y_1'}{\partial y} + 3 \dfrac{\partial X_3}{\partial y} Y_1 \right)$$

with initial conditions:

$\eta=C$: $\quad \dfrac{\partial X_3}{\partial y} = \dfrac{\partial X_3'}{\partial y} = \dfrac{\partial X_3''}{\partial y} = \dfrac{\partial Y_3}{\partial y} = 0, \qquad \dfrac{\partial Y_3'}{\partial y} = 1,$

$\eta=0$: $\quad \dfrac{\partial X_1}{\partial y} = \dfrac{\partial X_1'}{\partial y} = \dfrac{\partial X_1''}{\partial y} = \dfrac{\partial Y_1}{\partial y} = 0, \qquad \dfrac{\partial Y_1'}{\partial y} = 1.$

As stated previously, the partial derivatives with respect to x and y, that appears in the generalized correction equations (4.3.6) and (4.3.7) can be evaluated for these system of equations by integrating corresponding perturbation equations for the functions X_2', X_2'', Y_2, Y_2' and X_3', X_3'', Y_3, Y_3' respectively.

4.4 COMPUTATIONAL PROCEDURE

Usually, a boundary value problem is recasted as an initial value problem by the application of initial value method. Hence, the boundary layer and corresponding perturbation differential equations were rewritten as equivalent systems of first order differential equations. With proper initial guesses these were integrated by fourth order Runge-Kutta method for first three values and subsequently, Adams-Moulton predictor-corrector method was applied for one correction per step and a fixed increment. The corrections to the initial conditions Δx and Δy, which involves integrals of perturbation equations were determined. The process was repeated until the relative changes in the correction terms Δx and Δy are less than small pre-assigned values. Least square error was found out and tested with a small pre-assigned value. Previous guesses were corrected and the whole process was repeated until the desired accuracy was achieved, i.e. the least square error was found less than a small preassigned value.

The set of equations (4.2.1) and its' associated perturbation differentials equations (4.2.10) and (4.2.11), which are all third order in η, can be expressed as an equivalent system of fifteen first order differential equations with five equations coming from each set. Thirteen initial conditions in total are known, and two more conditions are to be guessed as initial estimates. These equations subject to the given numerical method, lends to the solution within desired accuracy.

The two other sets of coupled equations of the boundary layer respectively (4.3.9) and (4.3.10), with their corresponding perturbation differential equations (4.3.11), (4.3.12), (4.3.13) and (4.3.14) can be expressed as an equivalent system of twentyfour first order differential equation for each set. This is because each of the set of equations constitutes four dependent variables. Because of the involvement of the dependent variables X_1 and Y_1 in these equations, the number of initial conditions required to start with, are twenty eight and two conditions are guessed as before for each set. These initial conditions comprise all the given conditions required for the solution of equations (4.2.1), besides their own initial conditions. The solutions of equations (4.2.1) achieved by the method just outlined in the foregoing paragraphs, one had in particular good approximations for $X_1''(0)$ and $Y_1'(0)$. These values are then used as a complete information of the initial conditions required for the subsequent solutions of equations (4.2.10) and (4.2.11). The computational technique is exactly the same for these equations.

The method described above w_s programmed in FORTRAN IV on the DEC-10 Computer system. A general flow chart (Fig. 5.1) is presented for the main program adapted for all the system of equations. The flow chart of integration subroutine is presented in the Fig. 5.2. The FORTRAN listings of the method are enclosed in the appendix D. The integration formulas, used in the integration subroutine are also given in Appendix B.

4.5 RESULTS

With trial initial guesses 0.60 and -0.50 for $\dfrac{d^2X_1(o)}{d\eta^2}$ and $\dfrac{dY_1(o)}{d\eta}$ respectively, the set of equations (4.2.1) were integrated to a value of η_{edge} equal to 5.0 with initial step size of 0.05 for first four steps and subsequently raised to 0.1 for further steps. The preassigned values to compare least square error and increments in the initial estimates in the iteration process was kept 0.005 and 0.005 respectively. The final results are quoted in Table II. From this solution, we have in particular better approximations for $\dfrac{d^2X_1(o)}{d\eta^2} = 0.796700$ and $\dfrac{dY_1(o)}{d\eta} = -0.438100$. The least square error was found out to be 0.000972.

These values are utilized further to integrate the other two sets of equations (4.3.9) and (4.3.10) with the initial step size of 0.05 for first four steps and subsequently raised to 0.1 for further steps, for the same range of the value of η_{edge} i.e. 5.0. The initial values $\dfrac{d^2X_2(o)}{d\eta^2}$ and $\dfrac{dY_2(o)}{d\eta}$, corresponding to the solution of set of equations (4.3.9) are 0.004316 and -0.054234 respectively. The initial values of $\dfrac{d^2X_3(o)}{d\eta^2}$ and $\dfrac{dY_3(o)}{d\eta}$, corresponding to the solution of set of equations (4.3.10) are 0.008510 and 0.001510 respectively.

Solutions of equations (4.2.1) give approximately the velocity and temperature profile and their corresponding gradients in the adjacent boundary layer. These are plotted in figures 4.1 through (4.4) respectively. The perturbations to

the velocity and temperatures comes from the solutions of sets of equations (4.3.9) and (4.3.10) are plotted in figures (4.5) through (4.8).

The set of equation (3.2.27) are rewritten with the value of Grashoff number substituted from the data given in Appendix III as follows:

$$Nu_D = -(0.4262 \times 10^8)^{1/4} \left[\frac{dY_1(o)}{d\eta} + \bar{x}^2 \left(a \frac{dY_2(o)}{d\eta} + \frac{dY_3(o)}{d\eta} \right) \right]$$

(4.5.1)

for non-linear variation of the temperature,

$$Nu_D = -(0.4262 \times 10^8)^{1/4} \left[\frac{dY_1(o)}{d\eta} + \bar{x} \left(a \frac{dY_2(o)}{d\eta} + \frac{dY_3(o)}{d\eta} \right) \right]$$

(4.5.2)

for linear variation of temperature,

$$Nu_D = -(0.4262 \times 10^8)^{1/4} \left[\frac{dY_1(o)}{d\eta} + \bar{x} \frac{dY_3(o)}{d\eta} \right]$$

(4.5.3)

for isothermal case, when a = o.

Now, the dimensionless heat transfer, i.e. Nusselt number can be computed from the above equations utilizing the solution of boundary layer equations with respect to the dimensionless distance. Three cases of non-isothermal conditions are considered with three different values of a taken as 0.1, 0.5 and 2.5 respectively. The results are tabulated in table III and (IV) and corresponding plots are displayed in figures (4.9) through (4.12).

Numerical Results of the solution of system of equations (4.2.1)

η in steps	X_1	X_1'	X_1''	Y_1	Y_1'
0.0000	0.00000000	0.00000000	0.79670000	1.00000000	-0.43810000
0.0500	0.00081756	0.04043711	0.74118671	0.97218671	-0.43804183
0.1000	0.00381879	0.07474550	0.69898994	0.95619158	-0.43803864
0.1500	0.00834321	0.10237435	0.65795813	0.93721567	-0.43784128
0.2000	0.01463159	0.13996232	0.60620739	0.91240416	-0.43762473
0.3000	0.03151091	0.19616526	0.51877114	0.86868884	-0.43654889
0.4000	0.05358281	0.24390416	0.43697600	0.82512514	-0.43454978
0.5000	0.08002921	0.28375378	0.36100693	0.78181656	-0.43141548
0.6000	0.11009054	0.31630196	0.29095379	0.73888508	-0.42698513
0.7000	0.14306626	0.34214126	0.22682333	0.69646619	-0.42114964
0.8000	0.17831503	0.36186126	0.16855065	0.65470369	-0.41385131
0.9000	0.21525402	0.37604189	0.11600961	0.61374470	-0.40508199
1.0000	0.25355769	0.38524783	0.06902254	0.57373477	-0.39487986
1.1000	0.29215598	0.39002376	0.02736919	0.53481352	-0.38332460
1.2000	0.33123216	0.39089059	-0.00920497	0.49711079	-0.37053164
1.3000	0.37022025	0.38834237	-0.04098105	0.46074333	-0.35664531
1.4000	0.40880224	0.38284402	-0.06825965	0.42581234	-0.34183161
1.5000	0.44670511	0.37482967	-0.09135439	0.39240164	-0.32627084

Contd....

η in steps	X_1	X_1'	X_1''	Y_1	Y_1'
1.6000	0.48369763	0.36470171	-0.11058603	0.36057659	-0.31015044
1.7000	0.51958722	0.35283032	-0.12627725	0.33038375	-0.29365838
1.8000	0.55421671	0.33955349	-0.13874800	0.30185107	-0.27697737
1.9000	0.61922462	0.30997792	-0.15527107	0.24979029	-0.24372476
2.0000	0.64943731	0.29420041	-0.15991703	0.22623429	-0.22745360
2.1000	0.67805253	0.27806245	-0.16252492	0.20428601	-0.21158983
2.2000	0.70504399	0.26175475	-0.16335370	0.18389934	-0.19623740
2.3000	0.73040319	0.24544301	-0.16264469	0.16501875	-0.18148089
2.4000	0.75413705	0.22926970	-0.16062095	0.14758120	-0.16738602
2.5000	0.77626564	0.21335590	-0.15748702	0.13151799	-0.15400085
2.6000	0.79682014	0.19780318	-0.15342903	0.11675643	-0.14135718
2.7000	0.81584099	0.18269538	-0.14861511	0.10322137	-0.12947235
2.8000	0.83337620	0.16810042	-0.14319597	0.09083661	-0.11835100
2.9000	0.84947985	0.15407198	-0.13730573	0.07952602	-0.10798703
3.0000	0.86421075	0.14065112	-0.13106288	0.06921455	-0.09836541
3.1000	0.87763123	0.12786778	-0.12457126	0.05982902	-0.08946388
3.2000	0.88980615	0.11574224	-0.11792115	0.05129877	-0.08125462
3.3000	0.90080193	0.10428633	-0.11119043	0.04355615	-0.07370564
3.4000	0.91068583	0.09350470	-0.10444562	0.03653684	-0.06678207
3.5000	0.91952524	0.08339587	-0.09774298	0.03018015	-0.06044725
3.6000	0.92738716	0.07395320	-0.09112958	0.02442905	-0.05466363

η in steps	X_1	X_1'	X_1''	Y_1	Y_1'
3.7000	0.93433768	0.06516575	-0.08464424	0.01923032	-0.04939357
3.8000	0.94044163	0.05701909	-0.07831853	0.01453447	-0.04459990
3.9000	0.94576225	0.04949594	-0.07217756	0.01029568	-0.04024643
4.0000	0.95036092	0.04257681	-0.05524084	0.00647168	-0.03629832
4.1000	0.95429701	0.03624052	-0.06052298	0.00302362	-0.03272231
4.2000	0.95762768	0.03046461	-0.05503439	0.00208414	-0.02948695
4.3000	0.96040782	0.02522580	-0.04978181	0.00188415	-0.02656265
4.4000	0.96268994	0.02050029	-0.04476893	0.00140612	-0.02392178
4.5000	0.96362121	0.01806210	-0.04036812	0.00081752	-0.02272165
4.6000	0.96452418	0.01626402	-0.03999682	0.00067710	-0.02153864
4.7000	0.96595825	0.01249296	-0.03546437	0.00052166	-0.01938947
4.8000	0.96703748	0.00916327	-0.01116865	0.00045620	-0.01745237
4.9000	0.96780483	0.00251501	-0.00410525	0.00021854	-0.01570722
5.0000	0.96830094	0.00105468	-0.00126859	0.00010931	-0.01413561

LEAST SQUARE ERROR IS GIVEN AS = 0.000972

INITIAL ESTIMATES : $X_1''(0)$ = 0.796700

$Y_1'(0)$ = -0.438100

Table III

Nusselt Number for isothermal and non-isothermal

surface conditions on the fuel rod

$(a = 0.0, a = 0.1)$

Dimensionless distance from the origin $\bar{x}$	Isothermal surface condition $a = 0.0$	Nusselt number for non-isothermal surface conditions $a = 0.1$	
		Temperature vation of the form $\emptyset = 1+a\bar{x}$ (linear)	Temperature vation of the form $\emptyset = 1+a\bar{x}^2$ (non-linear)
0.0	35.399	35.399	35.399
0.1	35.393	35.609	35.415
0.2	35.387	35.822	35.479
0.3	35.380	36.043	35.587
0.4	35.371	36.256	35.739
0.5	35.365	36.472	35.933
0.6	35.357	36.688	36.171
0.7	35.349	36.905	36.451
0.8	35.341	37.119	36.774
0.9	35.328	37.336	37.142
1.0	35.315	37.551	37.551
1.1	35.301	37.788	38.004
1.2	35.286	37.982	38.512
1.3	35.269	38.198	39.151
1.4	35.249	38.414	39.881
1.5	35.223	38.629	40.491

Table IV

Nusselt number for non-isothermal

surface conditions on the fuel rod
(a = 0.5, a = 2.5)

Dimensionless distance from origin X $\bar{x}$	Nusselt Number			
	a = 0.5		a = 2.5	
	Temperature variation of the form $\emptyset = (1+a\bar{x})$ (linear)	Temperature variation of the form $\emptyset=(1+a\bar{x}^2)$ (Non-linear)	Temperature variation of the form $\emptyset = (1+a\bar{x})$ (linear)	Temperature variation of the form $\emptyset=(1+a\bar{x}^2)$ (Non-linear
0.0	35.399	35.399	35.399	35.399
0.1	35.837	35.437	36.517	35.505
0.2	36.281	35.571	37.649	36.845
0.3	36.724	35.793	38.777	36.409
0.4	37.168	36.103	39.905	37.198
0.5	37.611	36.502	41.033	38.213
0.6	38.055	36.990	42.161	39.452
0.7	38.498	37.567	43.288	40.120
0.8	39.942	38.232	44.416	42.612
0.9	39.385	38.986	45.544	44.529
1.0	39.829	39.829	46.672	46.672
1.1	40.273	40.760	47.800	49.040
1.2	40.716	41.781	48.928	51.634
1.3	41.160	42.889	50.055	54.454
1.4	41.603	44.081	51.183	57.499
1.5	42.047	45.328	52.311	60.770

$x_1' \sim \eta$ Curve

Dimensionless distance from the surface
of the fuel rod, η

Dimensionless Velocity x_1'
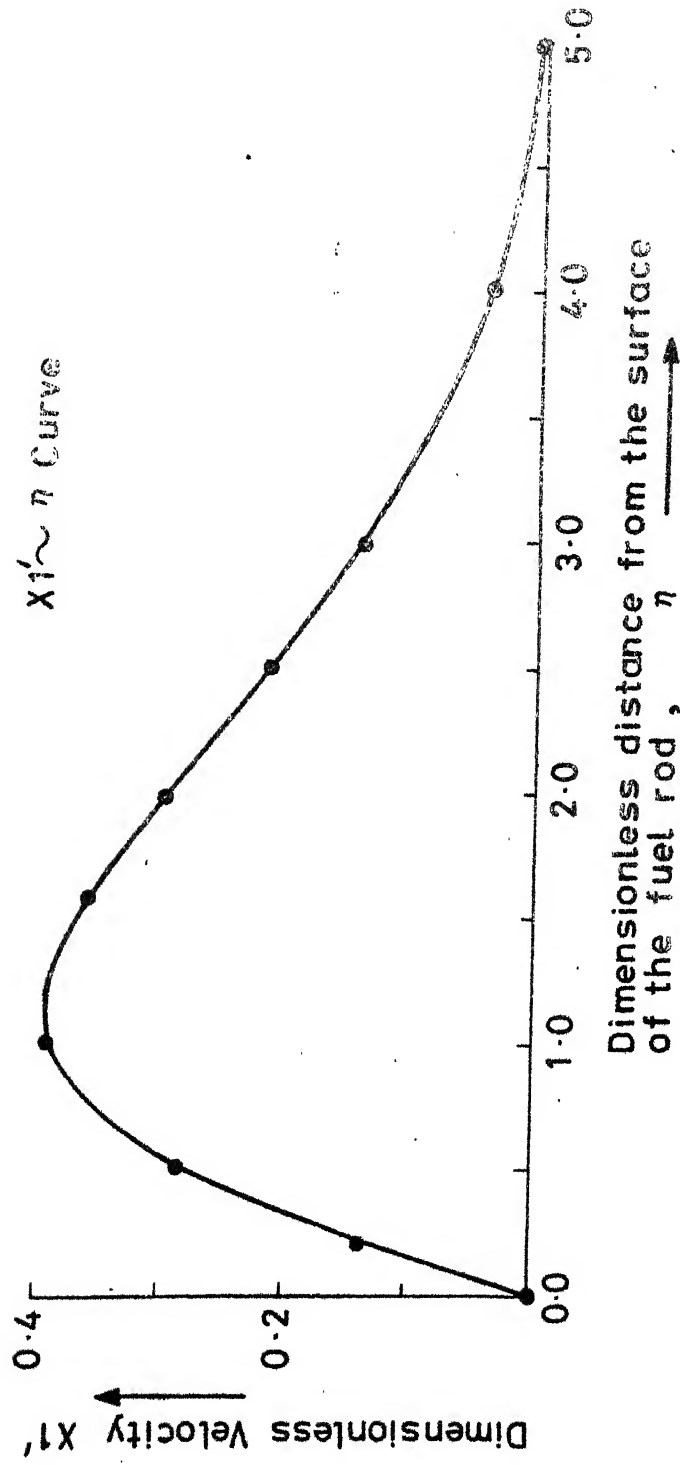
Fig. 4·1   Velocity Distribution in the boundary layer

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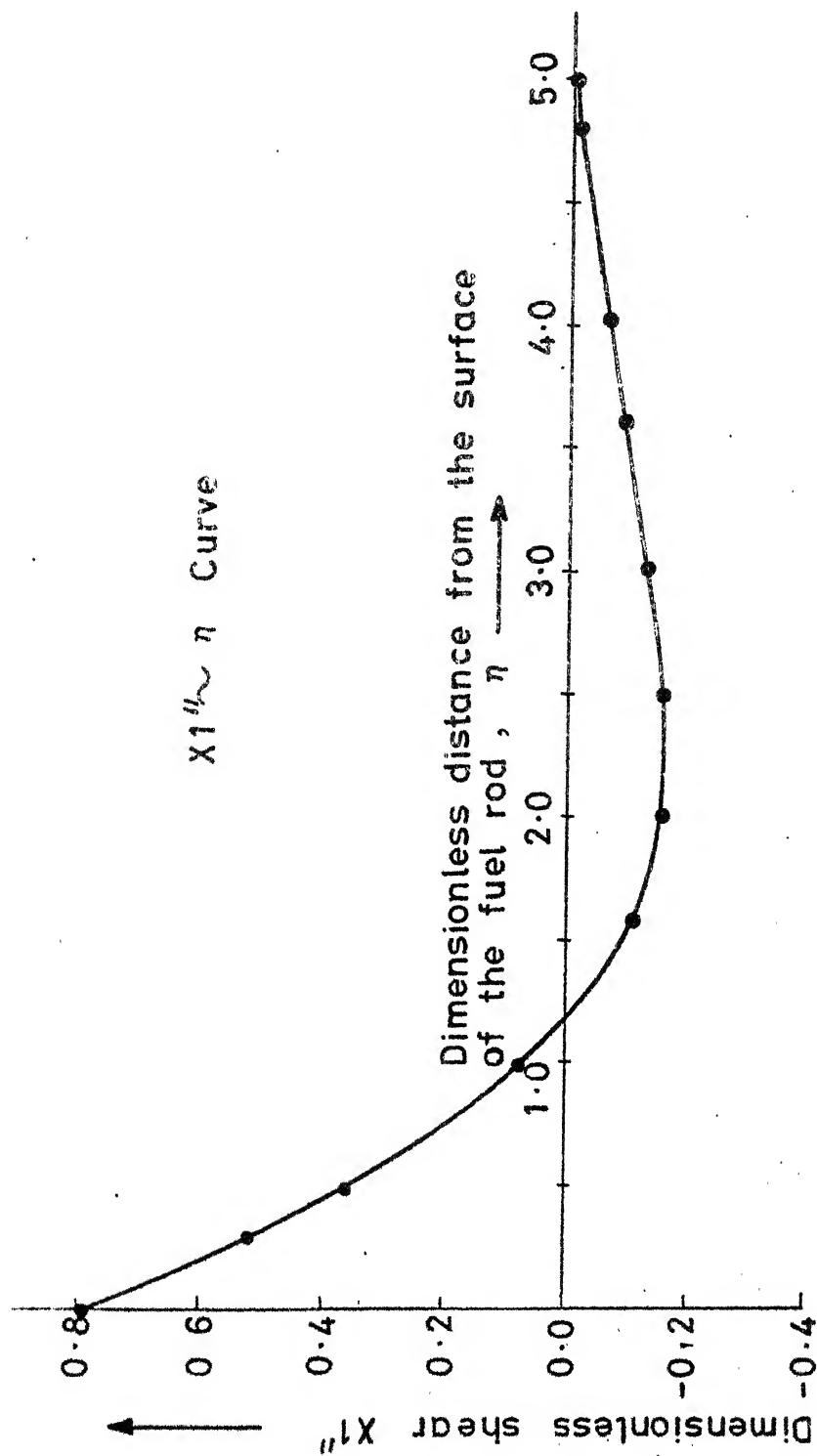
Fig. 4.2 Distribution of shear in the boundary layer
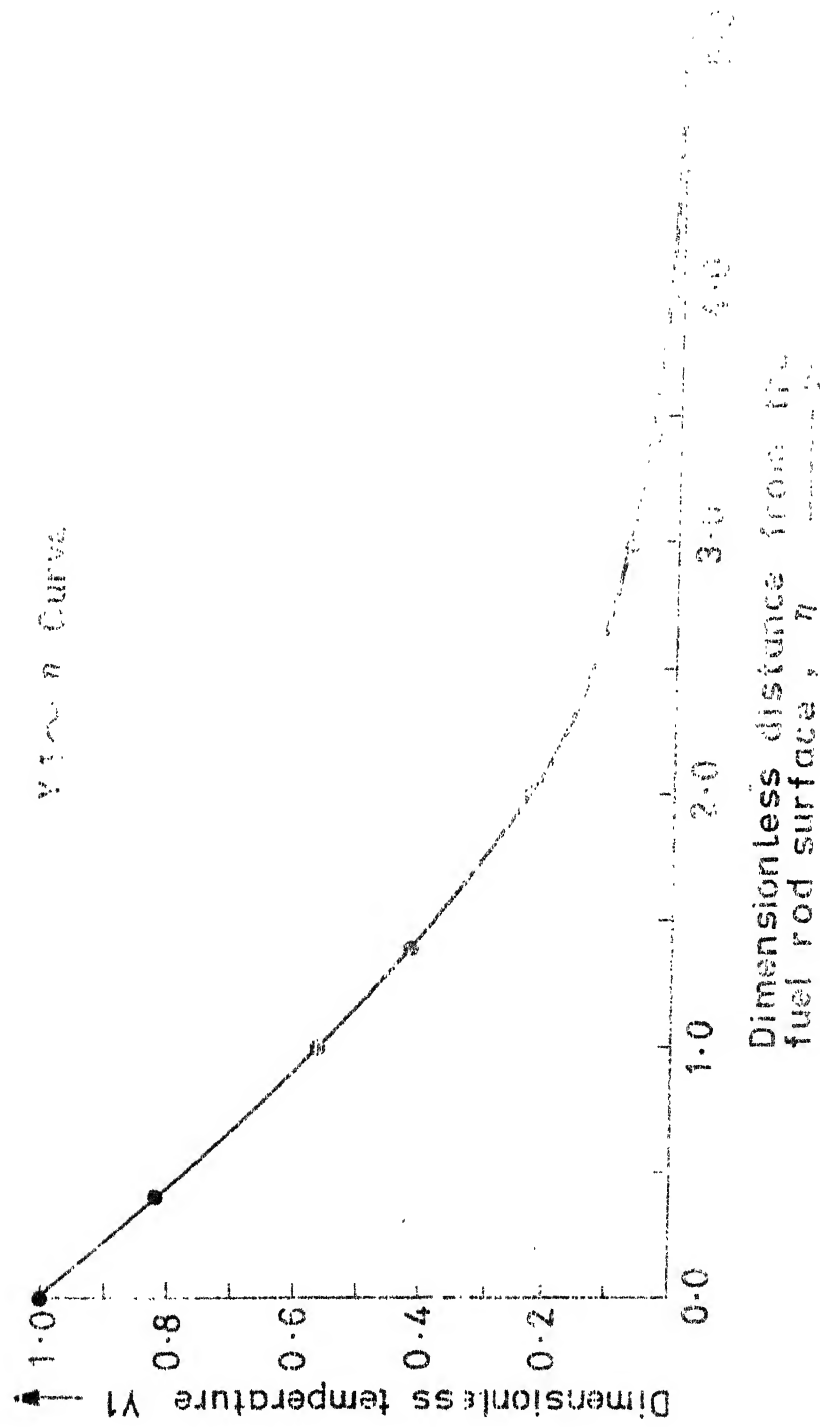
Fig. 4·3 Temperature distribution in the boundary layer
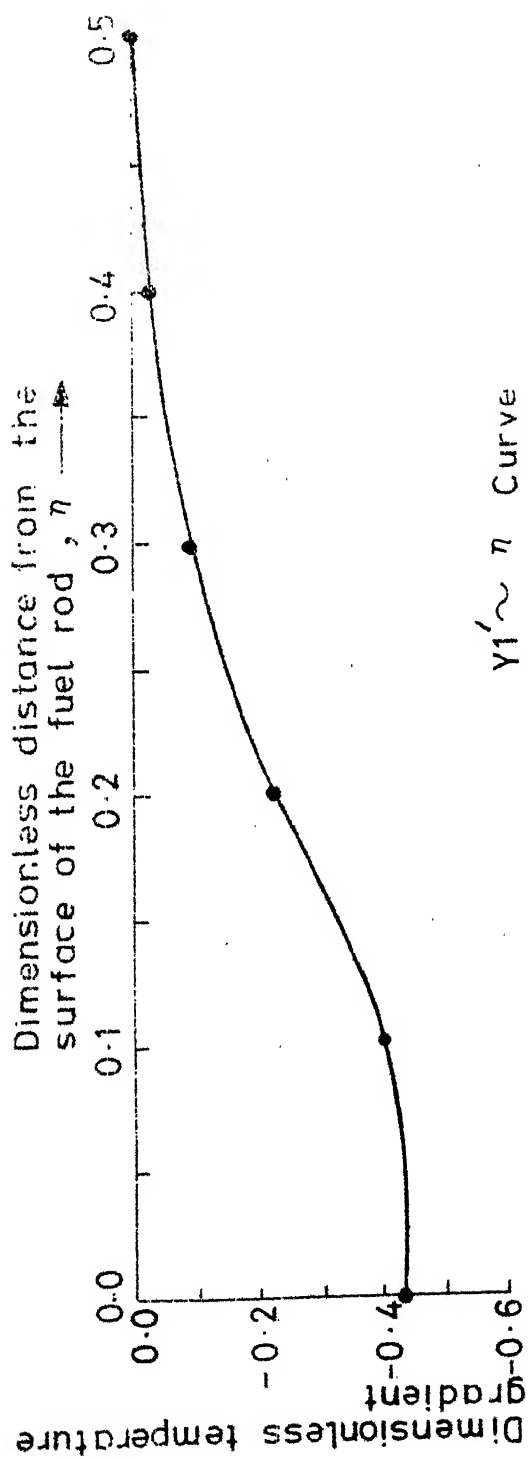
Fig. 4·4   Variation of temperature gradient

Fig. 4·5   Perturbations to the dimensionless velocity

$Y_2 \sim \eta$ Curve

Dimensionless distance from fuel rod surface, η
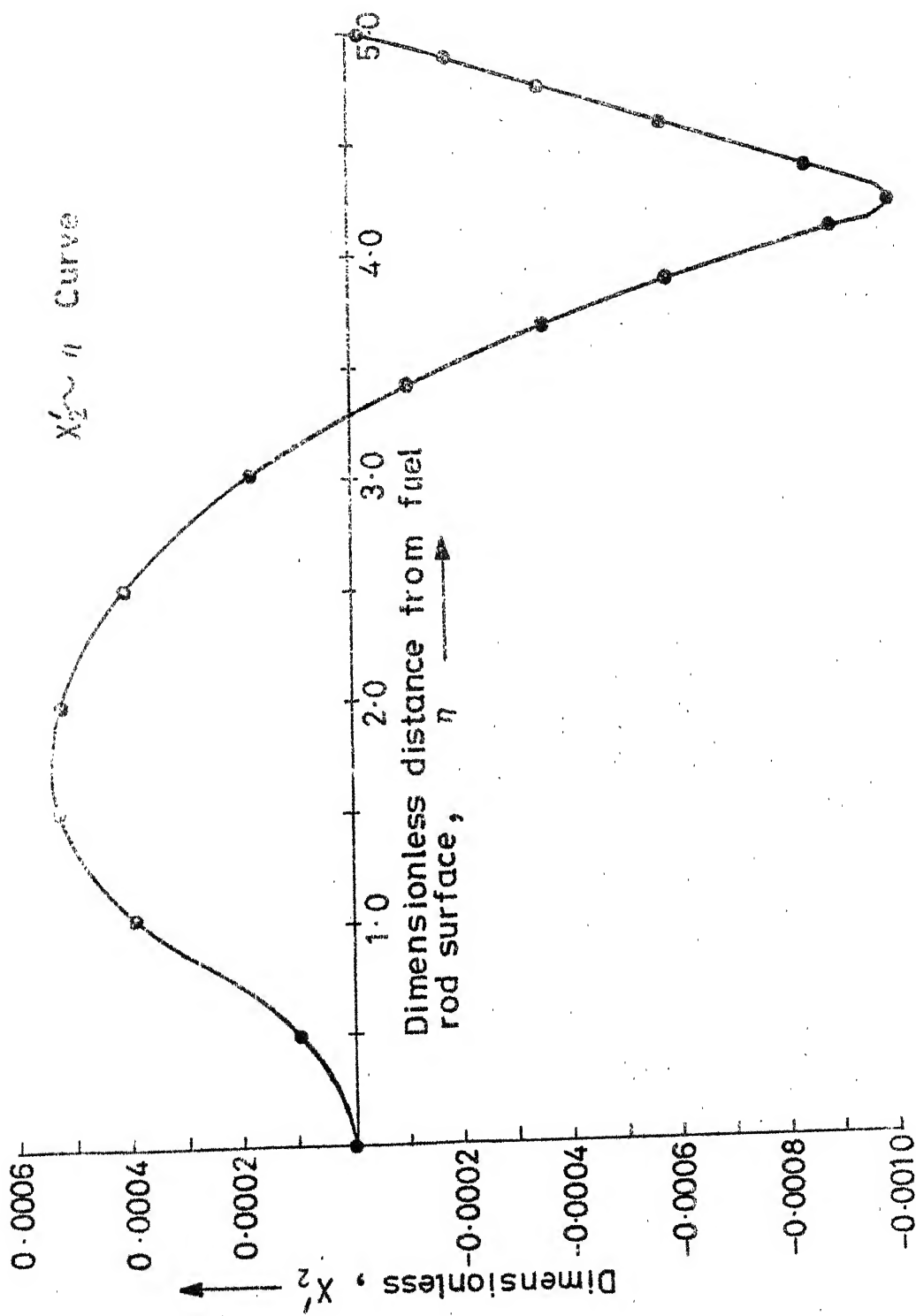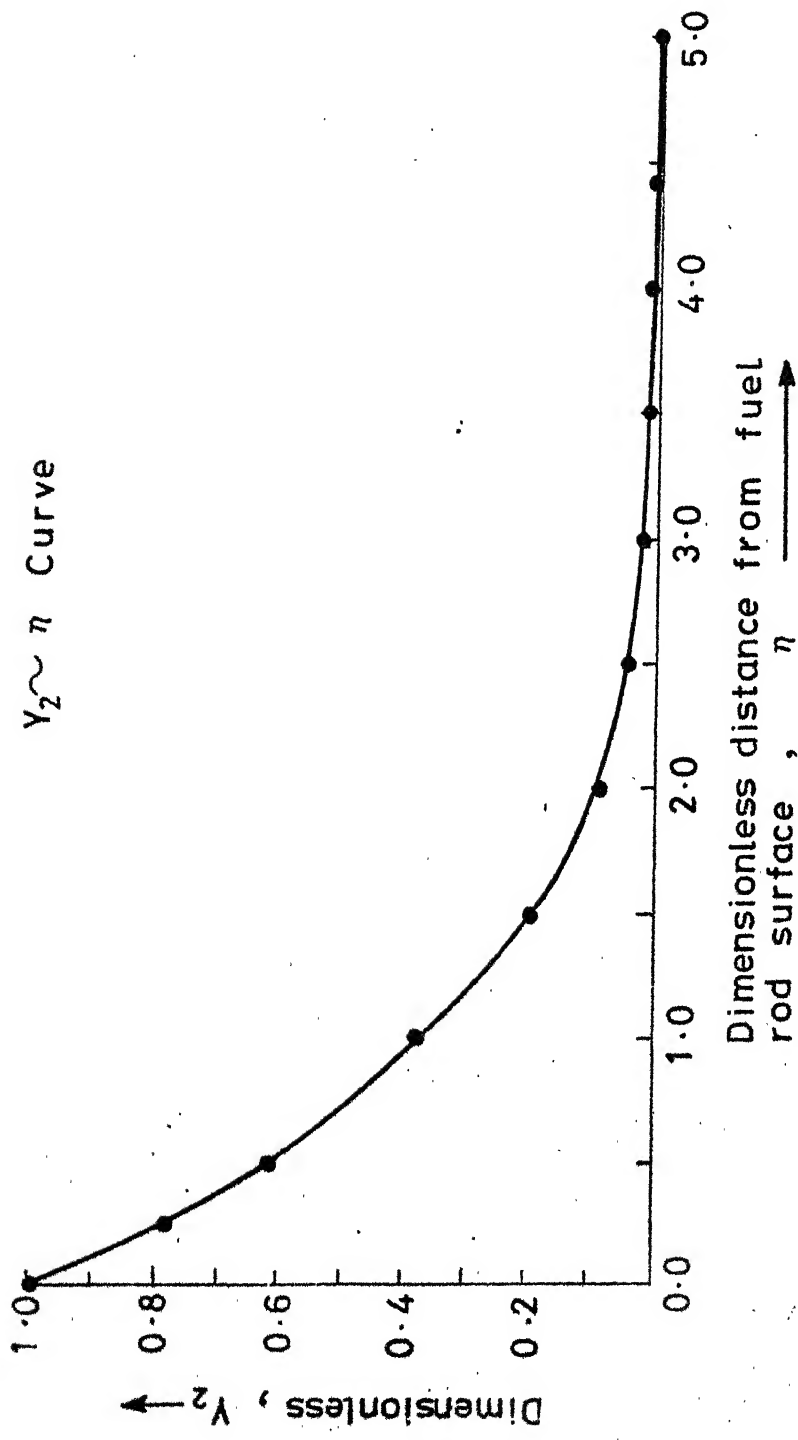
Dimensionless, Y_2

Fig. 4·6 Perturbations to the dimensionless temperature

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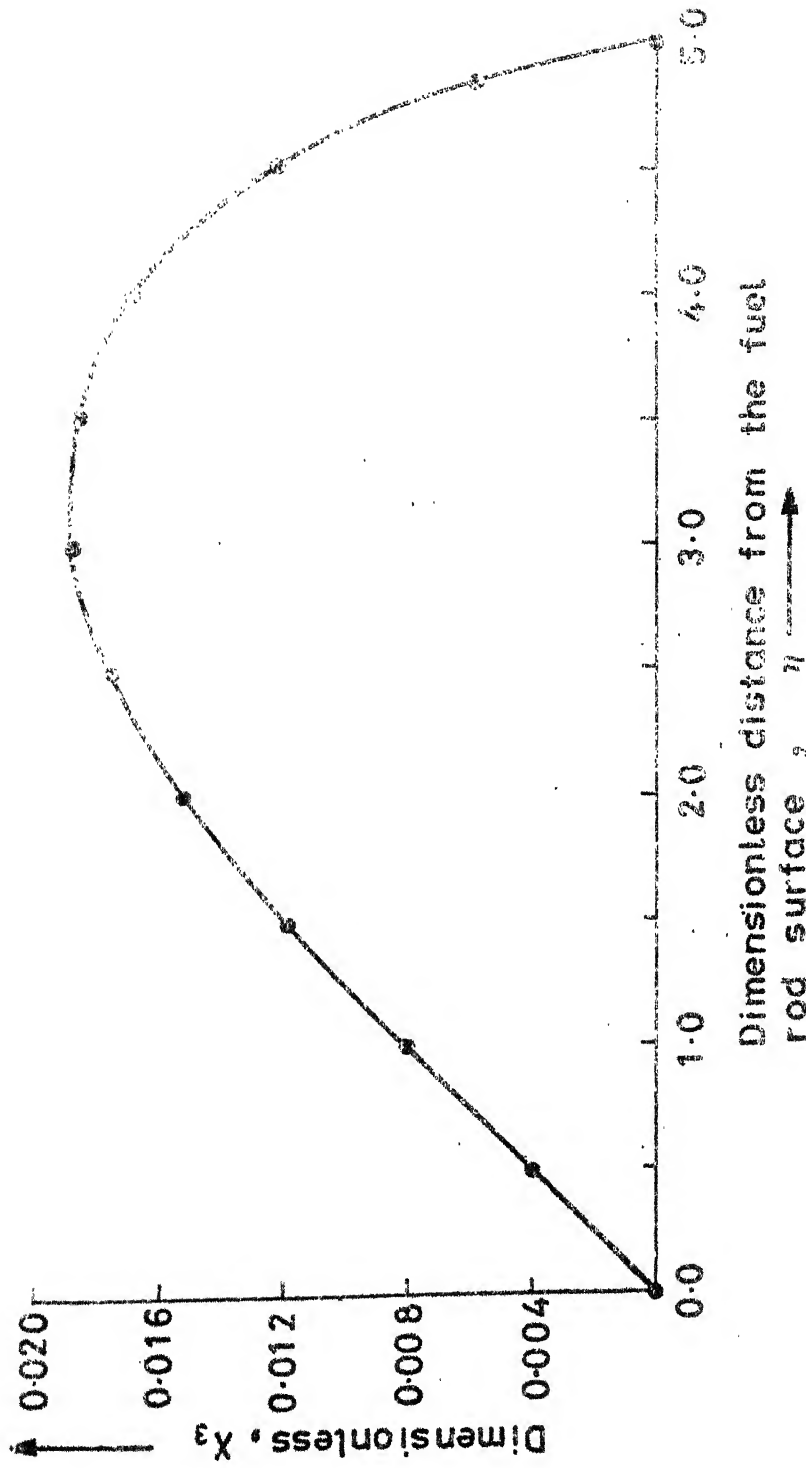
Fig. 4.7 Perturbations to the dimensionless velocity
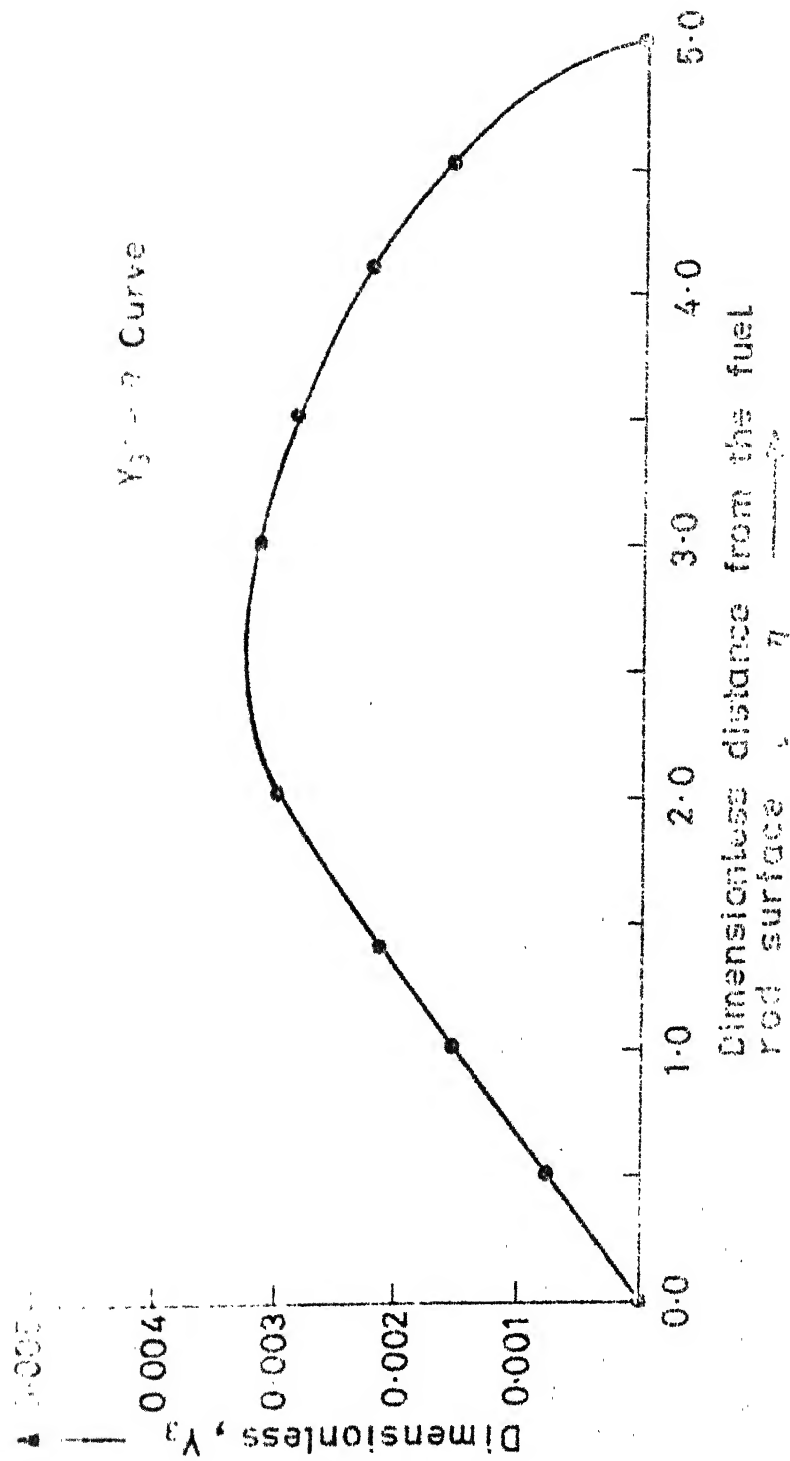
Fig. 4.8 Perturbations to the dimensionless temperature
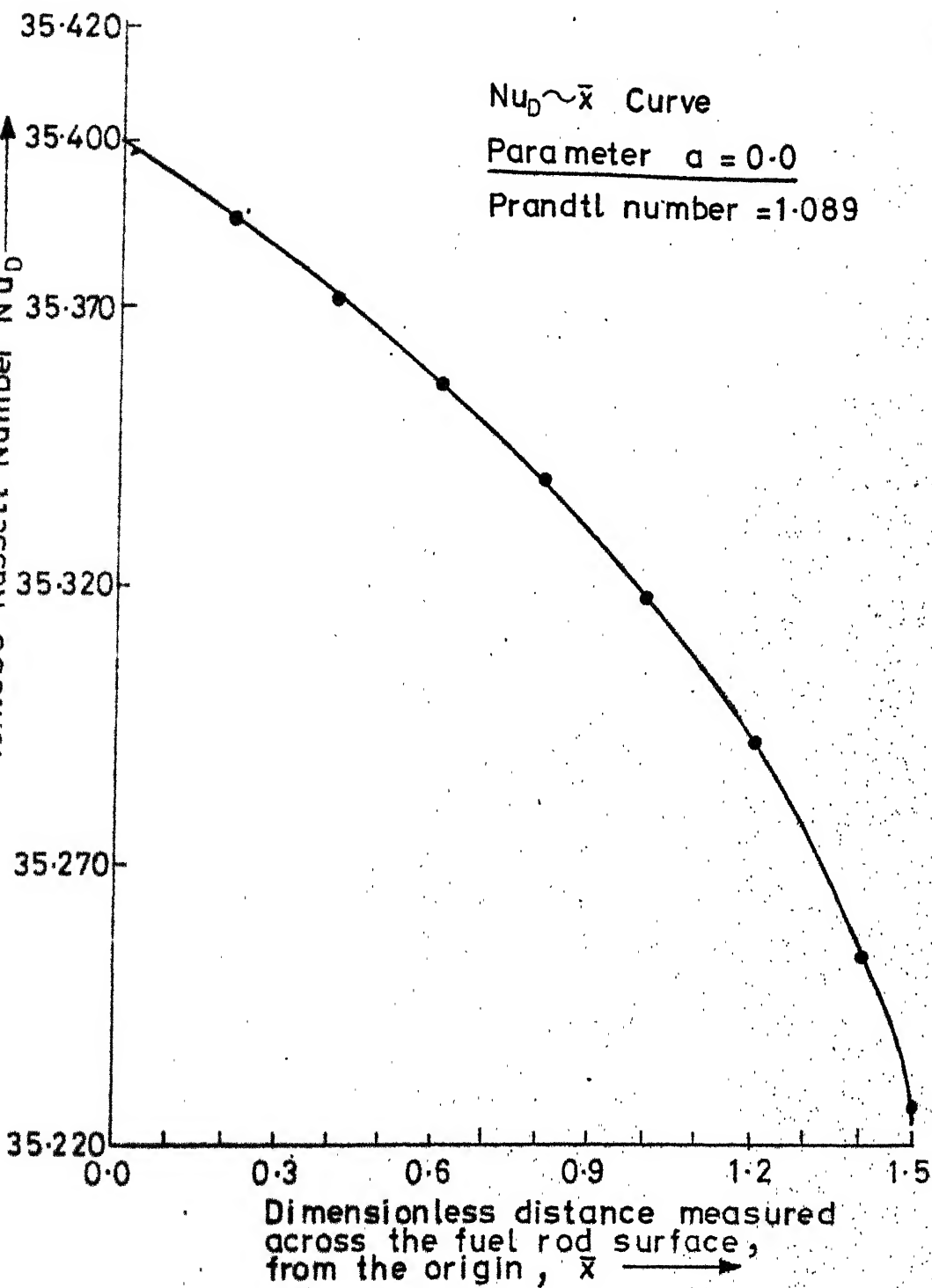
$y_3 - \eta$ Curve

Dimensionless distance from the fuel rod surface, η

Dimensionless, y_3

Fig. 4·9   Isothermal surface condition on the fuel rod

Dimensionless Nusselt number Nu_D

43.50

40.00

$Nu_D \sim \bar{x}$ Curve

Parameter $a = 0.1$

Prandtl number $= 1.089$

38.00

37.00

36.00

35.00

(i) Linear variation
$(T - T_\infty) = (T_{W_0} - T_\infty)(1 + a\bar{x})$

$(T - T_\infty) = (T_{W_0} - T_\infty)(1 + a\bar{x}^2)$
(ii) Nonlinear variation

0.0   0.3   0.6   0.9   1.2   1.5

Dimensionless distance measured across the
fuel rod surface from the
origin $\bar{x}$ ⟶
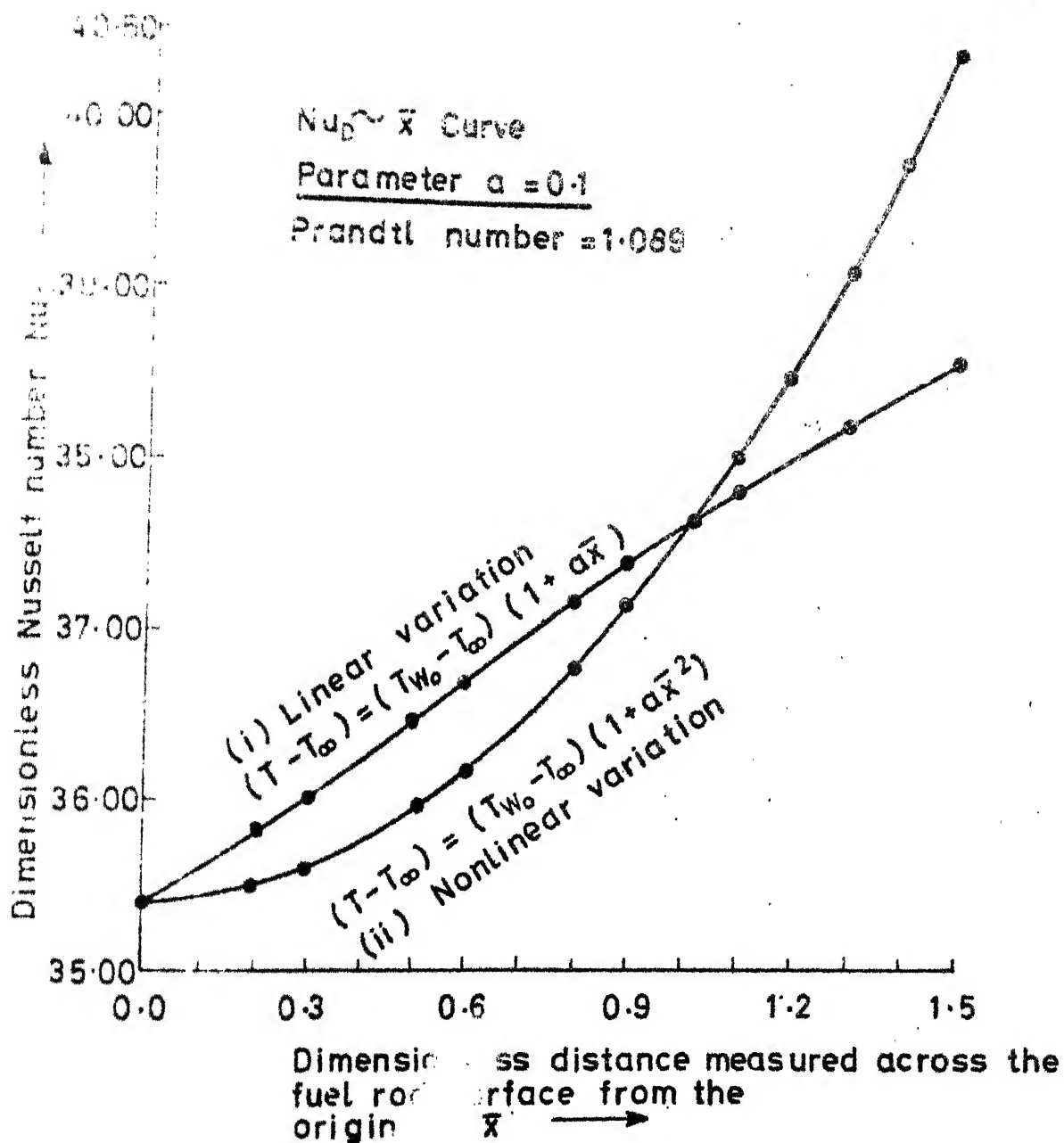
Fig. 4·10   Non thermal surface condition
on the fuel rod for $a = 0.1$
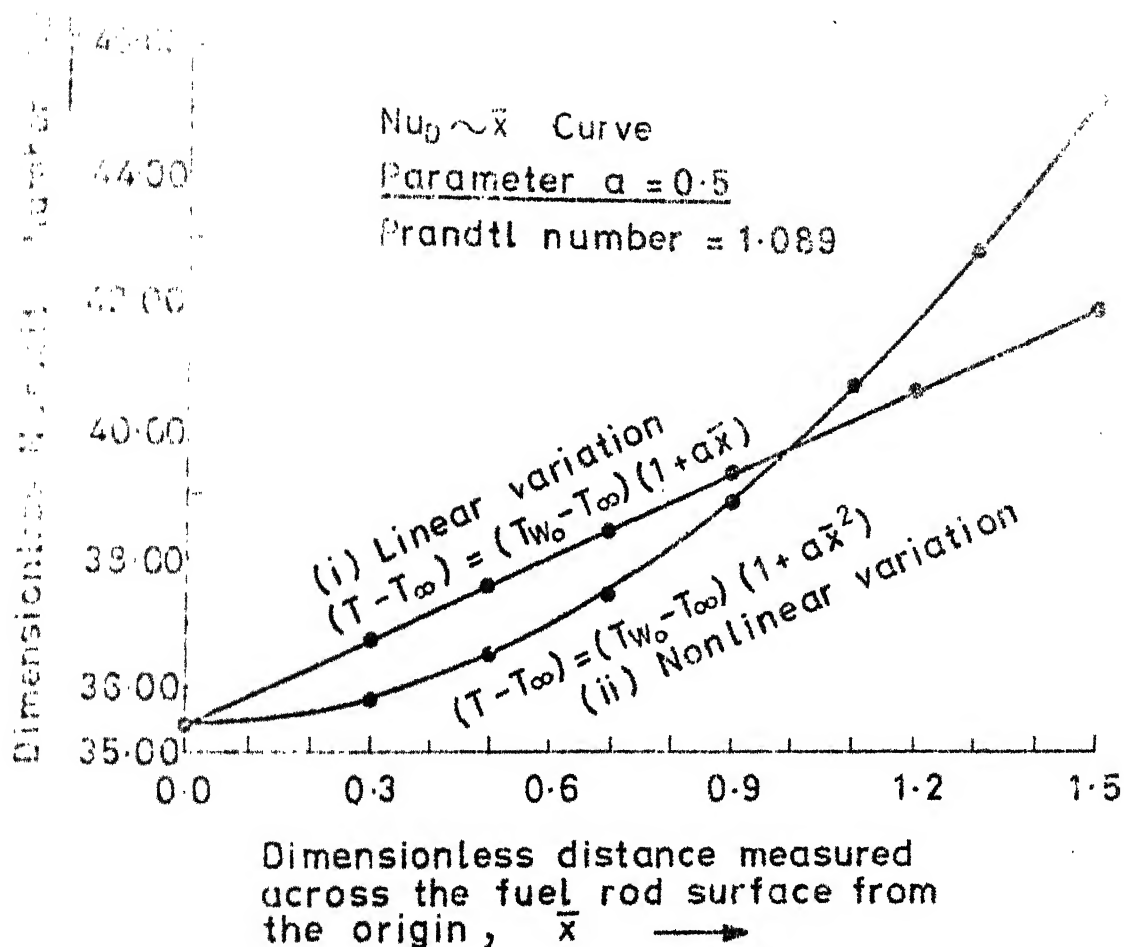
Fig. 4·11 Non-isothermal surface condition on the fuel rod for a = 0·5
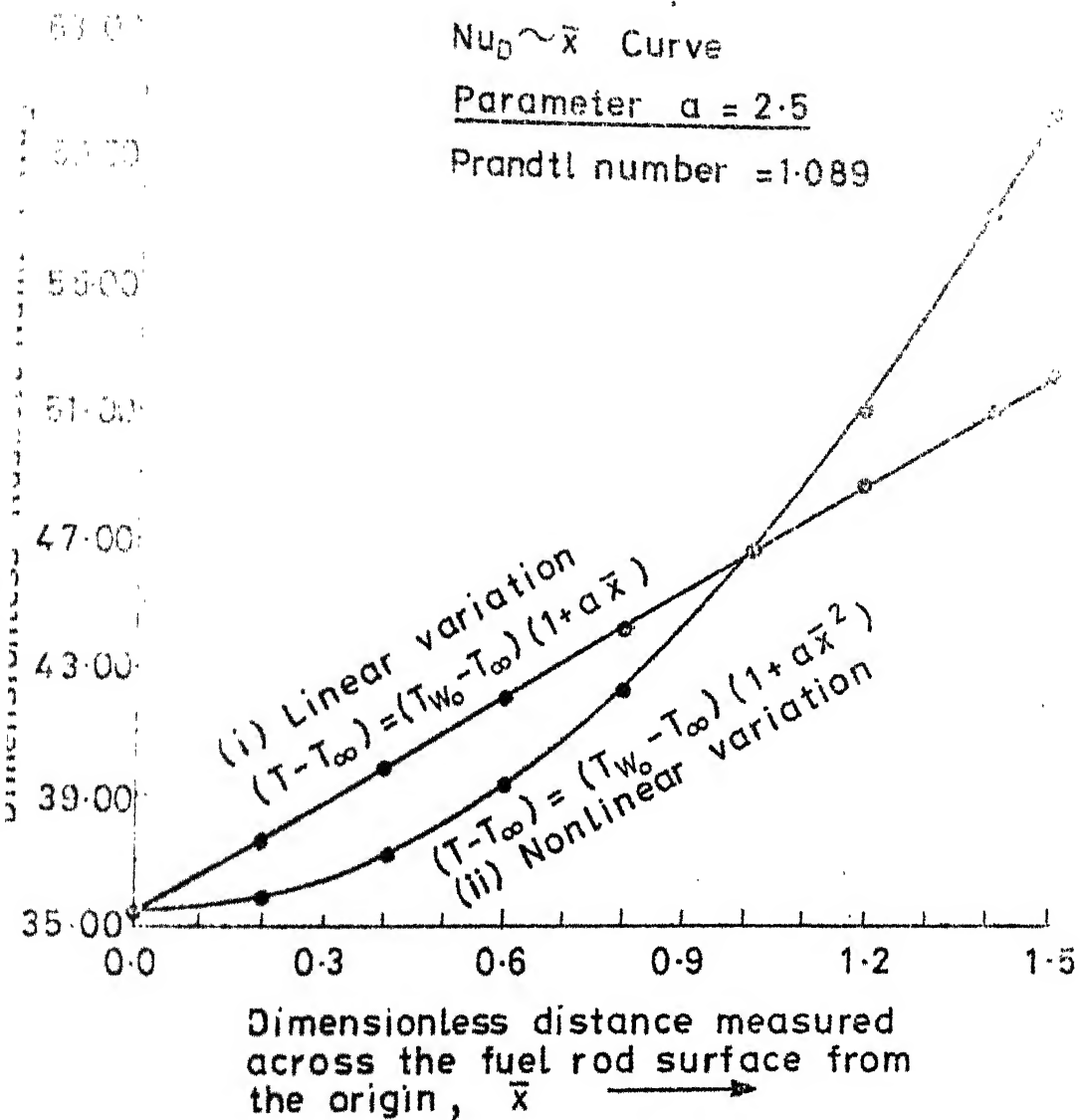
Fig. 4·12 Nonisothermal surface   condition
          on the fuel rod for a = 2·5

CHAPTER V

DISCUSSION OF RESULTS AND CONCLUSION

5.1 DISCUSSION OF RESULTS

The preponderance of the preceeding chapters are devoted to study the problem of transient flow and temperature behaviour associated with loss-of-flow accident in a CANDU Reactor core and principally following the transient, the thermosyphon (Natural convection) cooling effects are assessed in situations, when there is a variation of temperature on the fuel rod surface.

The results leading to a flow coastdown curve, Fig. 2.2, represents situations when pump flywheel inertia is large. Consequently, the pump will continue to circulate the fluid for a substantial length of time and the flow coastdown will be relatively slow. Following the total loss of pumping power, the driving pressure is found incapable of establishing natural circulation in the primary circuit.

The temperature distribution in the core under the transient condition, induced by loss-of-flow accident, assuming an initial sinusoidal heat flux distribution can be known from the solution of the transient heat conduction equation as cited in the equation (3.1.25). The steady state temperature distribution of the core follows from the same equation, when time is considered very large.

The natural convection thermosyphon results leading to velocity and temperature profiles show an asymptotic decrease as the edge of the boundary layer is approached. The thermosyphoning effects are attributed to the velocity distribution in the boundary layer. Nusselt number is calculated for natural convection around horizontal cylindrical objects by Hyman correlation as given in equation (1.3.10). It gives, $Nu_D = 37.38$.

The present heat transfer results as given in Table (IV) 3 III for isothermal and non-isothermal surface conditions are in close agreement with this value. Furthermore, the present results are approximately one fifteenth of forced convection Nusselt number (for coolant velocity 27.2 ft/sec) as shown in the text by K. Sri Ram [32]. The variation of surface temperature of the fuel rod in the manner $(1 + a\bar{x}^2)$ gives a sharp rise in the Nusselt number compared to the variation in the form $(1 + a\bar{x})$ for the values of $\bar{x} > 1$, as shown in Fig. 4.11.

5.2 CONCLUSION

On the basis of the analysis given in the foregoing chapters, the following principal conclusions can be drawn:

(1) The hydraulic modelling to determine the transient flow behaviour associated with the loss-of-flow accidents for coolant pumps designed with sufficiently large flywheels shows that, the flow coastdown will be slow. However, if the reactor trip is immediate, the after heat from the decay of fission products can be transported out of the core for a substantial length of time.

(2) The steady state temperature distribution in the core is attained approximately after 2 minutes.

(3) Since, the natural circulation can not be established in the primary flow circuit, thermosyphoning in the vicinity of each fuel rod provides immediate cooling. To investigate this phenomena, following Lighthills' [7] approach, the boundary layer differential equations for free convection flows around horizontal cylindrical fuel elements are formulated and solved by numerical methods. For non-isothermal conditions, the surface temperature of the fuel rod is assumed to be varied in two different manners, i.e. of the forms $(1 + a\bar{x})$ and $(1 + a\bar{x}^2)$. Suitable transformations are used so that the solutions will be independent of parameter a and hence, the method will be applicable for all reasonable values of a. Furthermore, it is found that the surface temperature variation has a significant effect on heat transfer.

5.3 SUGGESTIONS FOR FURTHER WORK

The present work can be extended in many ways as quoted below :

(a) The hydraulic transient analysis can be extended for very fast transients, where acoustical phenomena becomes important or to situations where transient boiling and rapid depressurization of the systems cuuse rapid time changes in the fluid density to occur. These can be accomplished by formulating appropriate mass, momentum and energy balance equations to such situations.

conceive of the pure convection process being relevant

for the random nature of the flow in the boundary layer

regime and possibly, neighbouring boundary layer inter-

actions becomes prevalent. This may also be investigated.

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APPENDIX A

Some aspects of solution of equation (2.2.10)

Equation (2.2.10) can be written as

$$t_\ell \frac{dy}{dt} + y^2 = \frac{1}{(1 + t/t_p)} \tag{A.1}$$

subject to initial condition,

$y = 1$ at $t = 0$, i.e. $y(0) = 1$, where, $y = \frac{W}{W_o}$

Equation (A.1) may be reduced to the form

$$t_\ell \left(\frac{d}{dt} + Z^2\right) - f(t) = 0 \tag{A.2}$$

by the substitution $y = Zt_\ell$, where $f(t) = \frac{1}{(1+t/t_p)^2}$

Equation (A.2) is a special case of Riccati equation

$$P_o(t)\left[\frac{dz}{dt} + Z^2\right] + P_1(t)Z + P_2(t) = 0 \tag{A.3}$$

where, $P_o = t_\ell$, $P_1 = 0$ and $P_2 = -f(t)$

A theorem, Brand [19] states that the Riccati equation (A.3) and the associated linear equation

$$P_o(t) \frac{d^2u}{dt^2} + P_1(t) \frac{du}{dt} + P_2(t)u = 0 \tag{A.4}$$

have corresponding solutions, which satisfy the equations

$$u(t) = \exp \int Z(t)dt, \quad Z(t) = \frac{1}{u} \frac{du}{dt}$$

By the above theorem, associated linear equation of (A.2) is found out to be

$$\frac{d^2u}{dt^2} - f(t)u = 0 \tag{A.5}$$

with transformed initial condition

$$\frac{1}{u(t)} \frac{du(t)}{dt} \bigg|_{t=0} = \frac{1}{t_\ell}$$

Equation (A.5) is of the form of Schrodingr equation. There ~~is~~ is no standard method of solution of this, equation and usually approximation theory is used for analytical solution.

APPENDIX B

INTEGRATION FORMULAS

Runge-Kutta 4th Order Formulas :

A pth order differential equations can be expressed as a system of p 1st order differential equations, i.e.

$$\frac{d^p y}{dx^p} = f(x, y, \frac{dy}{dx}, \frac{d^2 y}{dx^2}, \dots, \frac{d^{p-1} y}{dx^{p-1}}) \qquad (B.1)$$

It we take, $y_1 = y$, $y_2 = \frac{dy}{dx}$ and $y_p = \frac{d^{p-1} y}{dx^{p-1}}$, the system of equations, i.e.,

$$\frac{dy_1}{dx} = f_1 (x, y, \dots, y_p)$$

$$\frac{dy_2}{dx} = f_2 (x, y, \dots, y_p)$$

$$\frac{dy_p}{dx} = f_p (x, y, \dots, y_p)$$

Vectorially, $\frac{d\vec{Y}}{dx} = \vec{f}(x, \vec{Y})$ where, Y, f are column vectors. For a coupled equation of the form,

$$\frac{dy}{dx} = f(x, y, z) \text{ and } \frac{dz}{dx} = g(x, y, z)$$

the 4th order Runge-Kutta method is given as :

$$y_{n+1} = y_n + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$
$$z_{n+1} = z_n + \frac{1}{6} (\ell_1 + 2\ell_2 + 2\ell_3 + \ell_4) \qquad (B.2)$$

where,

$$K_1 = hf(x_n, y_n, z_n)$$

$$\ell_1 = hg\ (x_n,\ y_n,\ z_n)$$

$$k_2 = hf\ (x_n + \frac{h}{2},\ y_n + \frac{K_1}{2},\ z_n + \frac{\ell_1}{2})$$

$$k_3 = hf\ (x_n + \frac{h}{2},\ y_n + \frac{K_2}{2},\ z_n + \frac{\ell_2}{2})$$

$$\ell_2 = hg\ (x_n + \frac{h}{2},\ y_n + \frac{K_1}{2},\ z_n + \frac{\ell_1}{2})$$

$$\ell_3 = hg\ (x_n + \frac{h}{2},\ y_n + \frac{K_2}{2},\ z_n + \frac{\ell_2}{2})$$

$$K_4 = hf\ (x_n + h,\ y_n + K_3,\ z_n + \ell_3)$$

$$\ell_4 = hg\ (x_n + h,\ y_n + K_3,\ z_n + \ell_3)$$

where, h is the step size. Three values are first determined
from the above method to use them in Adam-Moulton predictor
corrector for better accuracy, as corrector an implicit itera-
tion technique is used for next predictor value.

Adams-Moulton Predictor Corrector Scheme:

Predictor : $y_{n+1}^{(0)} = y_n + \frac{h}{24}[55f(x_n,y_n,z_n) - 59f(x_{n-1},y_{n-1},$

$z_{n-1}) + 37f(x_{n-2},y_{n-2},z_{n-2}) - 9f(x_{n-3},\ y_{n-3},\ z_{n-3})]$

$$z_{n+1}^{(0)} = z_n + \frac{h}{24}[55g(x_n,y_n,z_n) - 59f(x_{n-1},y_{n-1},z_{n-1}) + 37g(x_{n-2},$$

$$y_{n-2},z_{n-2}) - 9g(x_{n-3},\ y_{n-3},\ z_{n-3})]$$

Corrector :

$$y_{n+1}^{(1)} = y_n + \frac{h}{24}[9f(x_{n+1},\ y_{n+1}^{(0)},\ z_{n+1}^{(0)}) + 19f(x_n,y_n,z_n)$$

$$- 5f(x_{n-1},\ y_{n-1},\ z_{n-1}) + f(x_{n-2},\ y_{n-2},\ z_{n-2})]$$

APPENDIX C
CANDU (RAPP) REACTOR DATA

The following Rajasthan Atomic Power Plant data of a CANDU type reactor is used here for the required calculations.

Each fuel bundle consists of 19 fuel rods of Natural UO_2.

(1)   Reactor inlet temperature $T_1 = 480^{\circ}F$

(2)   Reactor outlet temperature $T_2 = 560^{\circ}F$

(3)   Mean D_2O temperature $T = 515^{\circ}F$

(4)   Temperature at the centre of the core T_0 (calculated)
$$= 580^{\circ}F$$

(5) D_2O temperature at low power $= 504^{\circ}F$

(6)   Specific heat at constant pressure of the D_2O coolant C_p
$$= 1.7744 \ Btu/lb\text{-}F^{\circ}$$

(7)   Dynamic viscosity of the D_2O coolant $\mu = 0.2838$ lb/ft-hr

(8)   Thermal conductivity of D_2O coolant $K = 0.3066 \ \dfrac{Btu}{hr\text{-}F^{\circ}\text{-}ft}$

(9)   Reactor channel length $2\ell = 16.404$ ft

(10)   Mean D_2O density $\bar{\rho} = 52.91$ lb/ft^3

(11)   Mean D_2O density in the riser at outlet temperature (calculated) $\bar{\rho}_r = 52.91 \ \dfrac{lbm}{ft^3}$

(12)   Mean D_2O density in the downcomer at the inlet temperature $\bar{\rho}_d = 57.29$ lbm/ft^3

(13)   Total system losses (calculated) $\sum \Delta P = 1.413$ lbf/in^2

(14)   Driving pressure (calculated) $\sum \Delta P_d = 0.1827$ lbf/in^2

(15) Pump halftime for a typical recirculation pump,

$t_p = I/c\,\omega_0 = 5$ sec.

(16) Height of the riser (hotleg) and downcomer (coldleg) of the U-tube heat exchanger $= 6$ ft

(17) Prandtl number of pr $= \mu C_p/K = 1.089$

(18) Reynolds number of Re $= \rho vD/\mu = 3.68 \times 10^5$ (for the coolant velocity 27.2 ft/sec)

(19) Nusselt number (for the coolant velocity 27.2 ft/sec)

$$= Nu = 654$$

20) Heat transfer coefficient (calculated for the given Nusselt number) h $= NuK/D = 9300$ Btu/hr-ft^2-F

(21) Hydraulic diameter D $= 0.0216$ ft

(22) Steady state Grashoff number basing on the diameter of the fuel rod (calculated) $Gr_D = \dfrac{g\beta(T_w - T_\infty)D^3}{\nu^2}$

$$= 0.4262 \times 10^8$$

(23) Kinematic viscosity $\nu = \dfrac{\mu}{\rho} = 5.3638 \times 10^{-3}$ ft^2/hr

(24) Clad surface temperature $T_w = 573.8\,^\circ$F

(25) Thermal expansion coefficient $\beta = 4.0 \times 10^{-4}$ F$^{\circ -1}$

(26) Volumetric thermal source strength at the centre of the core $q_o''' = 1.648$ Btu/ft^3-sec

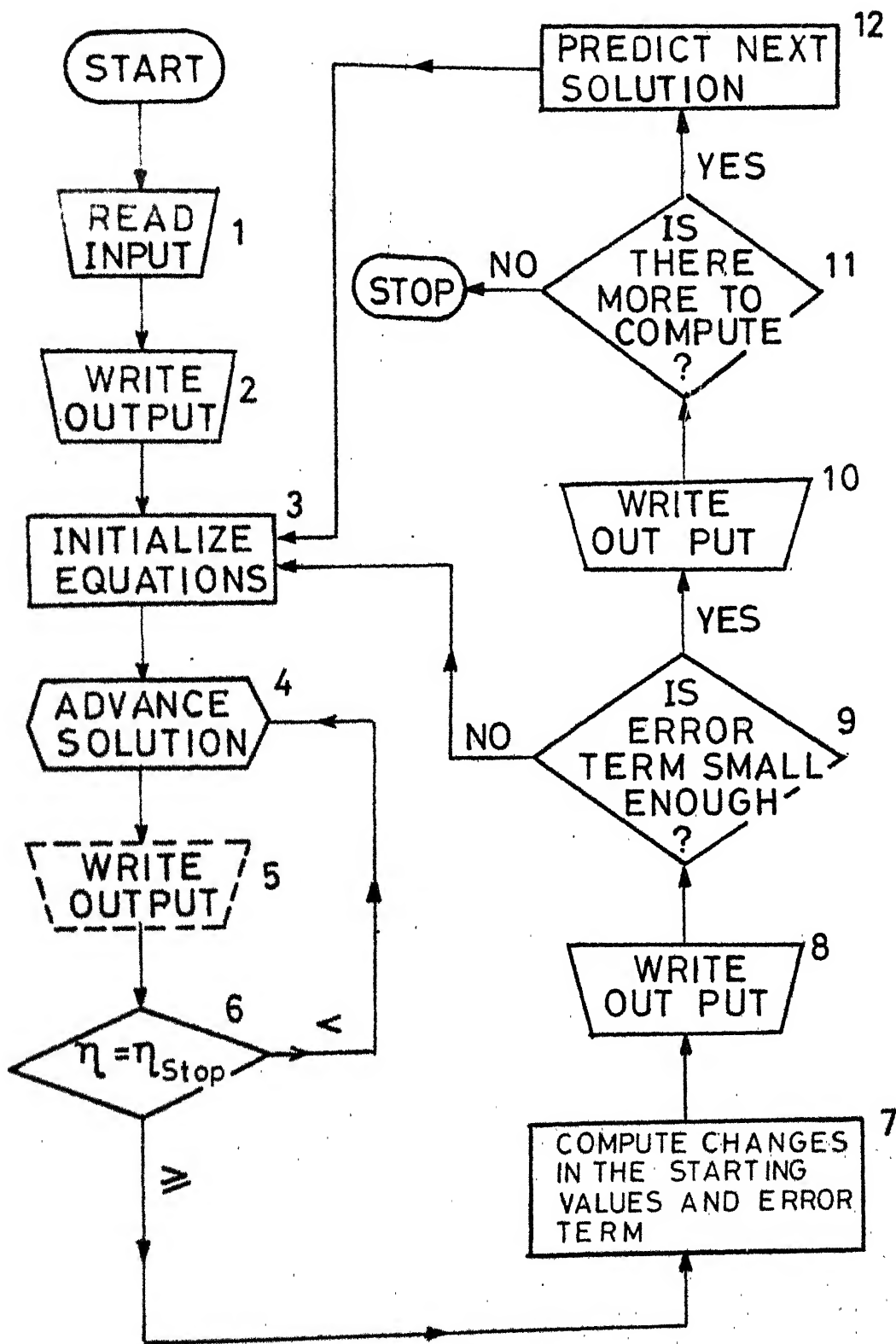(27) Diffusivity $\alpha = \dfrac{K}{\rho C_p} = 0.0032$ ft^2/sec.

Fig. 5·1 General flow chart for the main program applied to all the equations
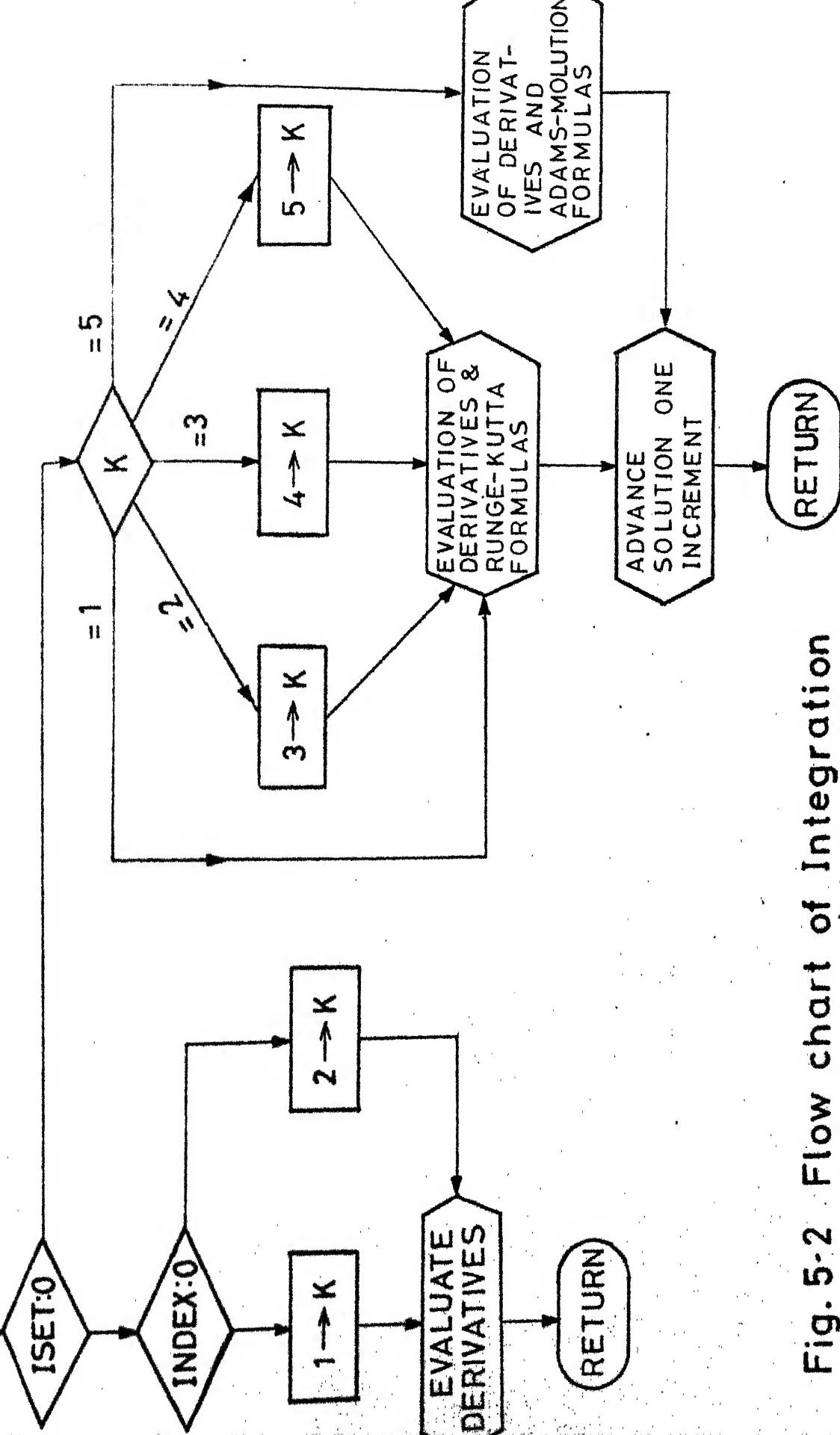
Fig. 5-2 Flow chart of Integration
Subroutine Adams

APPENDIX D

COMPUTER PROGRAM

```
C      LAMINAR M H D CONVECTION DIFFERENTIAL EQUATIONS ARE SOLVED
C      SOLUTION OF SYSTEM OF NONLINEAR DIFFERENTIAL EQUATIONS  1
C      THIS IS AN ASYMPTOTIC BOUNDARY VALUE PROBLEM FOR A NONLINEAR
C      COUPLED THIRD ORDER DIFFERENTIAL EQUATION
C      INITIAL VALUE METHOD IS APPLIED WITH NACHTSHEIM-SWIGERT
C      ITERATION SCHEME USING LEASTSQUARE ERROR MINIMIZATION
C      IN THE STATE PROBLEM
       BEGINING OF THE PROGRAM DEC
       DIMENSION P,R
       DIMENSION Y(30),DY(30),ETAEND(10),TEST(10)
       READ*,N,Y,T,PR,DELPR,LTEST
       READ*,P,(ETAEND(I),I=1,6)
       READ*,(TEST(I),I=1,N)
C      TEST(I), ACCURACY CRITERIA IMPOSED ON CORRECTION FOR
C      INITIAL CONDITIONS DELX AND  DELZ
       TYPE 101,N,Z,TEST
       TYPE 102,(ETAEND(I),I=1,N)
       TYPE 103,(TEST(I),I=1,N)
       TYPE 201,N,PR
       I=1
       STYPE=DELPR
C      'DELPR' IS THE TYPE INTERVAL WHICH WE KEEP SAME AS STEP SIZE
C      INITIAL SURFACE CONDITIONS ARE GIVEN BELOW
C      THESE ARE INITIAL CONDITIONS GIVEN AT THE CLAD SURFACE
C      EXCEPT, Y(3) AND Y(5) WHICH ARE TO BE GAUSSED
       ETA=0.0
       Y(1)=0.0
       Y(2)=0.0
       Y(3)=X
       Y(4)=1.0
       Y(5)=Z
       Y(6)=0.0
       Y(7)=0.0
       Y(8)=1.0
       Y(9)=0.0
       Y(10)=0.0
       Y(11)=0.0
       Y(12)=0.0
       Y(13)=0.00
       Y(14)=0.0
       Y(15)=1.0
C      LET US FIRST CALCULATE THE DERIVATIVES FOR ONLY FIRST VALUE
C      OF ETA AS A TRAIL, TO SEE THE RESULTS
       CALL ADAMS(15,N,ETA,0,Y,DY,1)
       TYPE 104,ETA,Y(1),Y(2),Y(3),Y(4),Y(5)
       TYPE 90
       TYPE 45
4      CALL ADAMS(15,N,ETA,1,Y,DY,1)
       TYPE 202,ETA,Y(1),Y(2),Y(3),Y(4),Y(5)
       IF(ETA.LT.STYPE)GO TO 6
       STYPE=STYPE+DELPR
6      IF(ETA.LT.ETAEND(I))GO TO 4
C      NACHTSHEIM-SWIGERT LEAST SQUARE ITERATION SCHEME IS
C      NOW APPLIED
       A11=Y(7)**2+Y(9)**2+Y(8)**2+Y(10)**2
       A12=Y(7)*Y(12)+Y(9)*Y(14)+Y(18)*Y(13)+Y(10)*Y(15)
       A21=A12
       A22=Y(12)**2+Y(14)**2+Y(13)**2+Y(15)**2
       R1=-(Y(2)*Y(7)+Y(4)*Y(9)+Y(3)*Y(8)+Y(5)*Y(10))
       R2=-(Y(2)*Y(12)+Y(4)*Y(14)+Y(3)*Y(13)+Y(5)*Y(15))
       DEM=A11*A22-A21*A12
C      CORRECTIONS FOR INITIAL CONDITIONS ARE GIVEN BELOW
C      CORRECTION IN X IS DELX AND Z IS DELZ
```

```
        DELX=(B1*A22-B2*A12)...
        DELZ=(B2*A11-B1*A21)...
        TYPE 50,DELX,DELZ
C       THE LEAST SQUARE ERROR IS GIVEN BELOW
        F=Y(2)**2+Y(3)**2+Y(4)**2+Y(5)**2
        TYPE 60,F
C       OLD INITIAL CONDITIONS ARE INCREMENTED BY DELX AND DELZ
C       RESPECTIVELY AND INTEGRATED AGAIN
        X=X+DELX
        Z=Z+DELZ
        TYPE 70,X,Z
50      FORMAT(/,5X,'INCREMENT IN INITIAL VALUE OF X=',2F15.8,/,5X,
       1 'INCREMENT IN INITIAL VALUE OF Z=',F15.8)
        IF(X.LT.1.24)STOP
        IF(ABS(DELX/X).GT.TEST(I).OR.ABS(DELZ/Z).GT.TEST(I)) GO TO 3
        IF(F.LT.TEST)GO TO 12
60      FORMAT(/,5X,'LEAST SQUARE ERROR IS GIVEN AS=',F8.6)
45      FORMAT(/,5X,'ETA IN STEPS',13X,'X1',15X,'X1*',15X,'X1**',
       1 15X,'Y1',15X,'Y1*')
        IF(I.EQ.M)STOP
        I=I+1
        GO TO 3
12      TYPE 203,F
        GO TO 3
90      FORMAT(/,5X,'THE INTEGRALS OF THE ORIGINAL DIFFERENTIAL EQN
       1 IS GIVEN BELOW',/,5X,'STAR(*) INDICATES FIRST DERIVATIVE


      1 DOUBLE STAR(**) INDICATES SECOND DERIVATIVE')
100     FORMAT(/,5X,'THE INITIAL VALUES TAKEN TO INTEGRATE THESE DIFFER
       1 ENTIAL EQUATIONS (ORIGINAL EQUATION)IS GIVEN BELOW',/,5X,
       1 'INITIAL VALUE OF INDEPENDENT VARIABLE TAKEN FROM THE CLAD
       1 SURFACE=',F15.8,/,5X,'X1(0)=',F15.8,5X,'X1*(0)=',F15.8,
       1 5X,'X1**(0)=',F15.8,/,5X,'Y1(0)=',F15.8,5X,'Y1*(0)=',F15.8)
101     FORMAT(5X,'INITIAL ESTIMATE X1**(0)=',F15.8,5X,/,5X,'Y1*(0)=',
       1 F15.8,/,5X,'ACCURACY CHOSEN FOR LEAST SQUARE ERROR
       1 MINIMIZATION=',F15.8)
70      FORMAT(/,5X,'NEW X=',F8.6,5X,'NEW Z=',F8.6)
102     FORMAT(/,5X,'THE VALUE OF ETA EDGE TAKEN ONE AFTER ANOTHER
       1 ,WHICH IS ESSENTIALLY THE RANGE OF INDEPENDENT VARIABLE
       1 IN THE TRAIL INTEGRATION=',4(F15.8,/))
103     FORMAT(/,5X,'ACCURACY DESIRED IN THE CORRECTION OF INITIAL
       1 CLAD SURFACE CONDITIONS IN THE LEAST SQUARE SOLUTION=',
       1 4(F15.6,/,43X))
201     FORMAT(/,5X,'STEP SIZE=',F15.8,/,5X,'PRANDTAL NO=',F15.8)
202     FORMAT(/,5X,6(F15.8,2X))
203     FORMAT(/,5X,'LEAST SQUARE ERROR=',F10.6)
        END
C       SUBROUTINE TO COMPUTE THE ORIGINAL AND CORESPONDING
C       PERTURBATION DIFFERENTIAL EQUATIONS
C       PERTURBATION EQUATIONS ARE USED TO DETERMINE THE PARTIAL
C       DERIVATIVES IN THE CORRECTION TERMS
C       THESE PARTIAL DERIVATIVES KNOWN AS CONVERGENCE DERIVATIVES
        SUBROUTINE DIFF(ETA,Y,DY)
        COMMON PR,X
        DIMENSION Y(30),DY(30)
C       FIRST ORIGINAL EQUATION IS WRITTEN AS FIVE EQUIVALENT
C       ORDINARY FIRST ORDER DIFFERENTIAL EQUATIONS
        DY(1)=Y(2)
        DY(2)=Y(3)
        DY(3)=Y(2)**2-Y(1)*Y(3)-Y(4)
        DY(4)=Y(5)
        DY(5)=-PR*Y(1)*Y(5)
C       NOW PERTURBATION EQUATIONS ARE WRITTEN AS EQUIVALENT FIRST
C       ORDER DIFFERENTIAL EQUATION
C       FIRST EQUATIONS CONTAINING X DERIVATIVES ARE WRITTEN
```

```fortran
      DY(6)=Y(7)
      DY(7)=Y(8)
      DY(8)=2.0*Y(2)*Y(7)-(Y(3)*Y(6)+Y(1)*Y(8))-Y(9)
      DY(9)=Y(10)
      DY(10)=-PR*(Y(5)*Y(6)+Y(1)*Y(10))
C     EQUATIONS CONTAING Y DERIVATIVES ARE NOW WRITTEN
      DY(11)=Y(12)
      DY(12)=Y(13)
      DY(13)=2.0*Y(2)*Y(12)-(Y(3)*Y(11)+Y(1)*Y(13))-Y(14)
      DY(14)=Y(15)
      DY(15)=-PR*(Y(5)*Y(11)+Y(1)*Y(15))
      RETURN
      END
C     INTEGRATION SUBROUTINE
C     SUBROUTINE TO INTEGRATE DIFFERENTIAL EQUATION OF THE TYPE
C     DY=F(X,Y) BY FOURTH ORDER RUNGE-KUTTA AND ADAMS MOULTON
C     FORMULAS
C     FIRST RUNGE-KUTTA METHOD APPLIED TO OBTAIN FIRST THREE VALUES
C     OF ADAMS-MOULTON PREDICTOR CORRECTOR WHICH IS SUBSEQUENTELY
C     APPLIED FOR BETTER ACCURACY
      SUBROUTINE ADAMS(N,H,X,ISET,Y,DY,INDEX)
      COMMON PR,K
      DIMENSION Y(30),DY(30),P(30),YR(30),DYLLL(30),DYLL(30),DYL(30),
     1DYR(30),C2(30),C3(30),C4(30)
      IF(ISET.GT.0)GO TO 6
      IF(INDEX.LT.0)GO TO 4
      K=2
      GO TO 5
1     K=1
5     CALL DIFF(X,Y,DY)
      RETURN
6     GO TO(10,7,8,9,11),K
7     K=3
      GO TO 10
8     K=4
      GO TO 10
9     K=5
C     FIRST RUNGE-KUTTA METHOD APPLIED
10    DO 1001 I=1,N
1001  P(I)=Y(I)+(H/2.0)*DY(I)
      CALL DIFF(X+H/2.0,P,C2)
      DO 1002 I=1,N
1002  P(I)=Y(I)+(H/2.0)*C2(I)
      CALL DIFF(X+H/2.0,P,C3)
      DO 1003 I=1,N
1003  P(I)=Y(I)+H*C3(I)
      CALL DIFF(X+H,P,C4)
      DO 1004 I=1,N
1004  YR(I)=Y(I)+(H/6.0)*(DY(I)+2.0*C2(I)+2.0*C3(I)+C4(I))  .
      CALL DIFF(X+H,YR,DYR)
      GO TO 12



C     ADAMS-MOULTON PREDICTOR CORRECTOR IS NOW APPLIED

11    DO 1101 I=1,N
1101  P(I)=Y(I)+(H/24.0)*(55.0*DY(I)-59.0*DYL(I)+37.0*DYLL(I)-
     119.0*DYLLL(I))
      CALL DIFF(X+H,P,DYR)
      DO 1102 I=1,N
1102  YR(I)=Y(I)+(H/24.0)*(9.0*DYR(I)+19.0*DY(I)-5.0*DYL(I)+DYLL(I))
      CALL DIFF(X+H,YR,DYR)
12    X=X+H
      DO 1201 I=1,N
      Y(I)=YR(I)
      DYLLL(I)=DYLL(I)
      DYLL(I)=DYL(I)
      DYL(I)=DY(I)
1201  DY(I)=DYR(I)
      RETURN
      END
```

```
LAMINAR FREE CONVECTION DIFFERENTIAL EQUATION 2
THE ITERATION PROCEDURE IS SAME AS GIVEN IN EQUATION 1
INITIAL CONDITIONS AND DIFFERENTIAL EQUATION SUBROUTINE
ARE ONLY MENTIONED
INITIAL CONDITIONS ARE GIVEN BELOW
ETA=0.0
Y(1)=0.0
Y(2)=0.0
Y(3)=X
Y(4)=1.0
Y(5)=Z
Y(6)=0.0
Y(7)=0.0
Y(8)=1.0
Y(9)=0.0
Y(10)=0.0
Y(11)=0.0
Y(12)=0.0
Y(13)=0.00
Y(14)=0.0
Y(15)=1.0
Y1(1)=1.0
Y1(2)=0.0
Y1(3)=0.67
Y1(4)=1.0
Y1(5)=-0.51
Y1(6)=0.0
Y1(7)=0.0
Y1(8)=1.0
Y1(9)=0.0
Y1(10)=0.0
Y1(11)=0.0
Y1(12)=0.0
Y1(13)=0.0
Y1(14)=1.0
Y1(15)=1.0
SUBROUTINE TO COMPUTE THE  ORIGINAL AND CORESPONDING
PERTURBATION DIFFERENTIAL EQUATIONS FOR SYSTEM OF EQUATIONS 2
PERTURBATION EQUATIONS ARE USED TO DETERMINE THE PARTIAL
DERIVATIVES IN THE CORRECTION TERSMS
THESE PARTIAL DERIVATIVES KNOWN AS CONVERGENCE DERIVATIVES
SUBROUTINE DIFF(ETA,Y,DY) FOR DIFFERENTIAL EQUATION 2
COMMON PR,K
DIMENSION Y(30),DY(30),Y1(30),DY1(30)
FIRST ORIGINAL EQUATION IS WRITTEN AS FIVE EQUIVALENT
ORDINARY FIRST ORDER DIFFERENTIAL EQUATIONS
DY1(1)=Y1(2)
DY1(2)=Y1(3)
DY(1)=Y(2)
DY(2)=Y(3)
DY(3)=-(Y1(1)*Y(3)-4.0*Y1(2)*Y(2)+3.0*Y1(3)*Y(1)-Y(4))
DY1(4)=Y1(5)
DY(4)=Y(5)
DY(5)=-PR*(Y1(1)*Y(5)-2.0*Y1(2)*Y(4)+3.0*Y(1)*Y1(5))
DY1(6)=Y1(7)
DY1(7)=Y1(8)
DY(6)=Y(7)
DY(7)=Y(8)
DY(8)=-(Y1(6)*Y(3)+Y1(1)*Y(8)-4.0*Y1(2)*Y(7)-4.0*Y1(7)*Y(2)
1+3.0*Y1(8)*Y(1)+3.0*Y1(3)*Y(6)-Y(9))
DY1(9)=Y1(10)
DY(9)=Y(10)
DY(10)=-PR*(Y1(1)*Y(10)+Y(7)*Y1(6)-2.0*Y1(7)*Y(4)
1-2.0*Y1(2)*Y(9)+3.0*Y(1)*Y1(10)+3.0*Y(6)*Y1(5))
DY1(11)=Y1(12)
DY1(12)=Y1(13)
DY(11)=Y(12)
DY(12)=Y(13)
DY(13)=-(Y1(11)*Y(3)+Y1(1)*Y(13)-4.0*Y1(2)*Y(12)-4.0*Y1(12)
1*Y(2)+3.0*Y1(13)*Y(1)+3.0*Y1(3)*Y(11)-Y(14))
DY1(14)=Y1(15)
DY(14)=Y(15)
DY(15)=-PR*(Y1(1)*Y(15)+Y(5)*Y1(11)-2.0*Y1(12)*Y(4)
1-2.0*Y1(2)*Y(14)+3.0*Y(1)*Y1(15)+3.0*Y(11)*Y1(5))
RETURN
END
```

```
C    ... CORRECTION DIFFERENTIAL EQUATION 3
C    THE ITERATION PROCESS IS SAME AS GIVEN IN EQUATION 1
C    DIFFERENTIAL EQUATION SUBROUTINE AND CORRESPONDING
C    ... ROUTINES ARE ONLY IDENTIFIED
C    ... ROUTINES ARE GIVEN BELOW.
      Y(1)=0.0
      Y(2)=0.5
      Y(3)=0.0
      Y(4)=A
      Y(5)=0.0
      Y(6)=0.0
      Y(7)=0.0
      Y(8)=1.0
      Y(9)=0.0
      Y(10)=0.0
      Y(11)=0.0
      Y(12)=0.0
      Y(13)=1.0
      Y(14)=0.0
      Y(15)=0.0
      Y(16)=0.0
      Y(17)=1.0
      Y(18)=-0.0
      Y(19)=0.0
      Y(20)=1.0
C
C    ... ROUTINE TO COMPUTE THE ORIGINAL AND CORRESPONDING
C    INFORMATION DIFFERENTIAL EQUATIONS FOR SYSTEM OF  EQUATIONS  3
C    INFORMATION EQUATIONS ARE USED TO DETERMINE THE PARTIAL
C    DERIVATIVES IN THE CORRECTION TERSMS
C    THESE PARTIAL DERIVATIVES KNOWN AS CONVERGENCE DERIVATIVES
      SUBROUTINE DY=F(ETA,Y,DY)
      DIMENSION Y(30),DY(30),Y1(30),DY1(30)
C    FIRST ORIGINAL EQUATION IS WRITTEN AS FIVE EQUIVALENT
C    FIRST ORDER DIFFERENTIAL EQUATIONS
      DY1(1)=Y1(2)
      DY1(2)=Y1(3)
      DY1(3)=Y1(4)

      (4)+Y(1)-4.0*Y1(2)*Y(2)+3.0*Y1(3)*Y(1)+Y(4)

      1(1)*Y(5)-2.0*Y1(2)*Y(4)+3.0*Y(1)*Y1(5))

      )*Y1(3)+Y1(1)*Y(8)-4.0*Y1(2)*Y(7)-4.0*Y1(7)*Y(2)
      )*Y1(3)+3.0*Y1(3)*Y(6)+Y(9)-0.166*Y1(9))

      )+Y1(4)+3.0*Y(6)*Y1(7)*Y1(6)-2.0*Y1(7)*Y(4)
      )+3.0*Y(6)+Y1(6))

      )+Y1(2)+Y1(3)+3.0*Y1(3)=4.0*Y1(2)*Y(12)-4.0*Y1(12)
      )+Y(14)-0.166*Y1(14))

      )+Y1(1)*Y(12)+Y1(2)*Y(15)-2.0*Y1(13)*Y(4)
      )+3.0*Y(6)*Y1(5))
      I=2
      DY(2)=-RA*Y(2)
      RETURN
      END
```

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